

Information Frictions and the Paradox of Price Flexibility *

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Abstract

The introduction of digital price tags may facilitate price adjustments and reduce the degree of nominal rigidity in the economy. Is this welfare-improving? To address this question, we build a multi-sector New Keynesian model with information frictions and dispersed beliefs. We show analytically in a static model and quantitatively in a dynamic model that increased price flexibility is welfare-deteriorating—the paradox of price flexibility. The presence of information frictions aggravates the paradox. Moreover, we study the optimal inflation index stabilization policy within our framework. The dispersed belief creates a novel channel through which the welfare cost of inflation in a sector is increasing in its price flexibility, and alters the optimal inflation index. A monetary policy that stabilizes the optimal inflation index mitigates the paradox.

Keywords: Dispersed belief, Information Frictions, Inflation Stabilization, Optimal Monetary Policy

JEL Classification: E31, E32, E52, E58

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1 Introduction

Electronic shelf labels (ESL) permits retailers to set prices digitally without any costs that would otherwise occur using paper price tags. Over the past decade, we have witnessed an expansion in the usage of digital price tags thanks to the growing affordability of ESL.¹ The introduction of digital price tags may facilitate price adjustment and reduce the degree of nominal rigidity in the economy. Is such technological progress welfare-improving? The thought experiment of a similar nature was first exploited by Keynes (1936) and later formalized by Long and Summers (1986). They postulated that an increase in price flexibility might increase output volatility.² While the recent trend of technological progress in price-setting has made this thought experiment likely to occur in the near future, its welfare consequence and implications for the conduct of monetary policy has received limited attention in the academic circle.

We address this research question in a multi-sector New Keynesian (NK) model with information frictions and dispersed beliefs. Empirical evidence supporting the presence of information frictions and dispersed beliefs are abundant, see, e.g., Coibion and Gorodnichenko (2012, 2015), Andrade et al. (2016) and Coibion et al. (2018) for a recent survey. We highlight a new channel—*dispersed belief channel* that is relevant to understand the welfare consequence of a reduction in nominal rigidity. The multi-sector feature of the model permits us to analyze the consequence of both an economy-wide and a sectoral reduction in nominal rigidities. The latter is relevant as the technological progress, such as the expansion of the ESL, might be merely a sectoral phenomenon rather than an economy-wide change. Moreover, the multi-sector model that we build allows us to

¹See the report by Global Market Insights: <https://www.globenewswire.com/news-release/2018/09/18/1572161/0/en/Electronic-Shelf-Label-ESL-Market-to-hit-1bn-by-2024-Global-Market-Insights-Inc.html>

²Bhattarai et al. (2018) and Galí (2013) revisited this issue recently.

revisit the optimal inflation index stabilization policy ([Aoki 2001](#), [Mankiw and Reis 2003](#), [Benigno 2004](#)). In particular, we show how monetary policy contributes to the welfare consequence of increased price flexibility, and draw policy recommendations in case such a thought experiment materializes.

To fully understand the welfare consequence of a change in price flexibility, we derive the welfare loss function around the perfect information steady-state and decompose it into four components. The first two components are proportional to inefficient dispersions in price and quantities within sectors arise due to price stickiness and dispersed beliefs. Such dispersions are inefficient because goods matter for households' utility symmetrically, and production technologies are identical. First, due to nominal rigidity such as staggered prices *à la* [Calvo \(1983\)](#), price dispersion arises across those that can reset prices and those that are staggered with old ones. We denote this as the *Calvo channel*. The welfare losses associated with this component are a hump shape function of price rigidity. Second, dispersed information that gives birth to imperfect common knowledge or dispersed beliefs creates another channel through which price dispersion arises. Firms have different assessments about the state of the economy due to information frictions. Therefore, in contrast to a standard model with perfect information, price dispersion emerges among those firms that reset prices. We denote this as the *dispersed beliefs channel*. The associated welfare losses increase monotonically in the degree of price flexibility. The third component is proportional to the output gap volatility, and the fourth is proportional to the relative price gap across sectors. Both tend to increase as the nominal friction increases. The aggregate effect of increased price flexibility on welfare is thus ambiguous ex-ante.

We derive the following results. First, in a static and symmetric two-sector model, we derive analytically the conditions under which an economy-wide increase in price flexi-

bility is welfare-deteriorating—the paradox of price flexibility. With perfect information, such a reduction in nominal friction is welfare-improving. However, in the presence of information frictions, the dispersed beliefs channel might dominate. Consequently, the paradox of price flexibility arises. The relative importance of the dispersed beliefs component as compared to other elements of the welfare loss function depends on three key parameters: the signal-noise ratio that characterizes how dispersed the agents’ beliefs are, and the elasticities of substitution across goods both within a sector and across sectors. Those elasticities determine, respectively, the magnitude of welfare losses generated by price dispersion within a sector and relative price gap across sectors.

Second, the paradox is more severe if the reduction in nominal frictions is merely a sectoral phenomenon. In our baseline analysis, we focus on an inflation-targeting central bank that fully stabilizes the Consumer Price Index (CPI), which is the principal mandate among many central banks in the world. Given this policy, increased sectoral price flexibility is detrimental to social welfare, even in the absence of information frictions. The result is driven by the across-sector spillover effect: increased flexibility in one sector, conditional on the same monetary policy, makes price in the other sector more volatile. The introduction of information frictions renders the paradox of price flexibility aggravated due to the dispersed beliefs channel. Those results also hint on the sub-optimality of a CPI stabilization policy.

We study the design of optimal inflation index stabilization policy, emphasizing the role of the dispersed beliefs. The objective is to find the price/inflation index, constructed by the weighted average of sectoral prices/inflations, associated with the lowest welfare losses. The presence of information frictions brings a new insight into the design of an optimal price index stabilization policy. [Aoki \(2001\)](#) and [Benigno \(2004\)](#) stress the importance of constructing an optimal price/inflation index that is proportional to sectoral

nominal frictions. A sector with a relatively more flexible price deserves a smaller weight as the welfare cost of inflation in this sector, which arises through the Calvo channel, is lower. The dispersed belief channel that we emphasize dampens the results arising from the Calvo channel. Through the former, the same level of inflation causes much more price dispersions (among price resetting firms) in that sector if its nominal rigidity is reduced. Therefore, ignoring the dispersed beliefs channel would lead to a policy recommendation that under-reacts to inflation of a relatively more flexible price sector. Moreover, due to dispersed beliefs, the optimal weight that a fully flexible price sector receives is nontrivial. In other words, in contrast to [Aoki \(2001\)](#)'s advice, our results suggest that the central bank should not focus on core inflation solely.

Does an optimal inflation stabilization policy resolve the paradox of price flexibility? An economy-wide change in nominal frictions does not alter the optimal inflation index, and therefore, the optimal inflation index stabilization policy does not play a role in this case.³ When a reduction in nominal frictions is asymmetric across sectors, the optimal price index stabilization policy mitigates the paradox of price flexibility. After a reduction in nominal frictions in one sector, the central bank reacts optimally by assigning a higher weight on the price in the second sector at the cost of higher volatility in the first sector. With perfect information, the benefit strongly dominates the cost, and aggregate welfare losses are reduced. Therefore, the policy mitigates the paradox substantially. However, in the presence of information frictions, an additional source of cost arises. A more volatile price in the first sector is associated with a higher price dispersion among price resetting firms due to the dispersed beliefs. This adds additional challenge for a central bank to resolve the paradox/improve social welfare.

³In fact, in the symmetric case where nominal rigidities in both sectors are the same, the optimal inflation index coincides with the CPI, as it is shown by [Mankiw and Reis \(2003\)](#) and [Benigno \(2004\)](#).

We extend our static model into a dynamic stochastic general equilibrium model with nominal rigidity and dispersed beliefs. Our findings survive in a dynamic model. Under reasonable calibrations, with perfect information, an economy-wide reduction in nominal rigidity is welfare-improving only if the reduction is sufficiently large: a drop of average price duration by two quarters (the average duration of a price drops from four quarters to two quarters). Because of information frictions, the paradox of price rigidity arises irrespective of the magnitude of a reduction in nominal rigidity.⁴ This result holds under an optimal inflation index stabilization policy as the optimal inflation index coincides with the CPI. When the reduction in price rigidity only occurs in one sector, a strong paradox arises if the central bank does not adjust its policy optimally. An optimal inflation index stabilization policy would mitigate the paradox. In this case, for it to be welfare-improving, the cut in nominal rigidity needs to surpass more than 1.5 quarters of reduction in average price duration if the information were perfect, and more than 2.5 quarters in the presence of information frictions.

Literature The theoretical framework is related to the literature that incorporates dispersed beliefs into business cycle models (e.g. [Lucas \(1972\)](#), [Woodford \(2001\)](#), [Nimark \(2008\)](#), [Lorenzoni \(2009\)](#), [Angeletos and Jennifer \(2009\)](#), [Hellwig and Venkateswaran \(2009\)](#), [Angeletos and La'O \(2011\)](#); [Angeletos and La O \(2013\)](#), [Huo and Takayama \(2015a,b\)](#), [Melosi \(2016\)](#), [Angeletos et al. \(2016\)](#), [Angeletos and Lian \(2018\)](#), [Huo and Pedroni \(2019\)](#)).⁵ We contribute to this literature by extending a NK model with dispersed beliefs to a multi-sector framework and derive different components of welfare loss function explicitly. Moreover, we focus on the implications of interactions of nominal rigidity and informa-

⁴We do not consider the extreme case when price becomes fully flexible.

⁵See [Mankiw and Reis \(2002\)](#), [Sims \(2003\)](#), [Mackowiak and Wiederholt \(2009\)](#) for other applications of information frictions for business cycle analysis.

tion frictions for social welfare.

Our paper contributes to the literature that studies the implications of a reduction in nominal rigidity. The idea that an increase in price flexibility may increase output volatility dates back to [Keynes \(1936\)](#), was formalized by [Long and Summers \(1986\)](#), and recently it is revisited by [Bhattarai et al. \(2018\)](#). [Galí \(2013\)](#) concludes that a reduction in wage rigidity improves welfare only if the central bank reacts to inflation sufficiently aggressive in a closed-economy New Keynesian model featuring both price rigidity and wage rigidity. Following similar reasoning, in an open economy with the fixed exchange rate ([Galí and Monacelli \(2016\)](#)) or a closed economy when the monetary policy is constrained by the Zero Lower Bound ([Amano and Gnocchi \(2017\)](#) and [Billi et al. \(2018\)](#)), a labor market reform that results in a more flexible wage is not necessarily welfare-improving. Our contribution with respect to this literature is to emphasize the role of information frictions or dispersed beliefs in the welfare analysis of a changing nominal rigidity. With this, we introduce a new channel that was previously ignored in the literature: the dispersed belief channel. We show that a more flexible price is welfare-deteriorating even if i) it does not increase output volatility ii) monetary policy strongly reacts to the inflation.

Lastly, this paper contributes to the literature that studies the optimal inflation index: e.g., [Erceg, Henderson, and Levin \(2000\)](#), [Aoki \(2001\)](#), [Mankiw and Reis \(2003\)](#), [Benigno \(2004\)](#), [Huang and Liu \(2005\)](#), [Anand, Prasad, and Zhang \(2015\)](#), [Basu and Leo \(2016\)](#) and [Zhang \(2018\)](#). Most of those papers rely on nominal rigidities, except for [Anand et al. \(2015\)](#) who study the role of financial frictions; and [Zhang \(2018\)](#) who studies the interaction between markup and nominal rigidity and its implication for the optimal inflation index stabilization policy. We contribute to this line of research by investigating the role of information frictions.

The remainder of the paper is organized as follows. Section 2 presents a static model and derive the analytical solution to shed light on the role of dispersed beliefs for the implications of a reduction in the nominal rigidity for social welfare and the optimal inflation index stabilization policy. Section 3 builds a dynamic model. Section 4 conducts quantitative analysis. Section 5 concludes.

2 Model

In this section, we present a static model to illustrate our mechanism, and we derive analytical solutions.

2.1 The Economy

The static model is a two-sector economy with imperfect common knowledge among firms. Sectors differ in the degree of nominal rigidity and are subject to sector-specific shocks.

Household We assume a representative household with complete information. A household is composed of two types of workers. Each type of them is endowed with a sector-specific skill and supply their labor in that sector. There is no labor mobility across sectors. The household makes sectoral labor supply and aggregate consumption decision to maximize the expected utility with the following utility function:

$$U(C, L) = \frac{C^{1-\sigma}}{1-\sigma} - \sum_{k=1}^2 L_k,$$

subject to the budget constraint:

$$PC = \sum_{k=1}^2 W_k L_k + \sum_{k=1}^2 \Pi_k + T,$$

where C is the consumption, P is the price of the consumption goods, L_k denotes the amount of labor that a household member supplies to sector k , W_k is the nominal wage in sector k , π_k is the profits generated by firms in sector k and T captures the net transfer from government. The final good that enters household's utility function is defined as:

$$C = \left[\sum_{k=1}^2 n_k^{1/\eta} C_k^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)}$$

where n_k is the size of the sector k with n_k summing up to one and η measures the degree of substitution among goods across sectors. The sector k 's output is a CES aggregate of different varieties C_{ki} :

$$C_k = \left[n_k^{-1/\epsilon} \int_0^1 C_{ki}^{(\epsilon-1)/\epsilon} di \right]^{\epsilon/(\epsilon-1)},$$

where ϵ measures the degree of substitution among varieties within a sector k . The implied aggregate prices are:

$$P = \left[\sum_{k=1}^2 n_k P_k^{1-\eta} \right]^{1/(1-\eta)} \quad P_k = \left[\int_0^1 p_{k,i}^{1-\epsilon} di \right]^{1/(1-\epsilon)}$$

where P denotes the economy-wise aggregate price and P_k denotes the aggregate price at sector k .

Solving the consumer's problem yields the following demand functions:

$$C_k = \left(\frac{P_k}{P}\right)^{-\eta} n_k C \quad C_{ki} = \left(\frac{p_{k,i}}{P_k}\right)^{-\epsilon} \frac{1}{n_k} C_k. \quad (2.1)$$

Firms There is a continuum of monopolistic competitive firms in each sector. Both sectors share the production function of the same functional form:

$$Y_{ki} = e^{a_k} L_{ki}, \quad (2.2)$$

where the (log) productivity a_k is unknown. We assume a fraction $(1 - \theta_k)$ of firms in sector k are free to adjust prices, and the remaining θ_k of firms are not allowed to reset price. The reoptimizing firms would choose the optimal price to maximize the expected profits:

$$\max_{P_{k,i}^*} E \left\{ P_{k,i}^* Y_{ki} - \frac{W_k Y_{ki}}{e^{a_k}} | \mathcal{I}_{ki} \right\} \quad (2.3)$$

subject to the demand functions 2.1, production function 2.2 and market clearing conditions $C_{ki} = Y_{ki}$, $C_k = Y_k$, $C = Y$. $\mathcal{I}_{ki}$ is the information set of firm i in sector k . We will explain it carefully later.

Solving the profit maximization problem of a firm i in sector k yields the following loglinearized optimal price setting rule:

$$p_{k,i}^* = E[p + \sigma(y - y^N) + u_k | \mathcal{I}_{ki}] \quad (2.4)$$

All variables are in log deviations from their initial values. p is the aggregate price defined as $p = \sum_1^k p_k$. p_k is the sector level price denoted as $p_k = \int_i p_{k,i}$. As a fraction θ_k of firms

are staggered to (log) price at initial level zero in sector k , therefore $p_k = (1 - \theta_k)p_k^*$ where $p_k^* = \int_i p_{k,i}^*$. y^N is the natural output under the assumption that all firms are allowed to adjust their prices with perfect information, which is linear in the productivities a_k ⁶, u_k is proportional to the relative sectoral productivity, in particular: $u_1 = n_2(a_2 - a_1)$ and $u_2 = n_1(a_1 - a_2)$. Let x denotes the output gap ($y - y^N$) and we can rewrite equation (2.4) as:

$$p_{k,i}^* = E[p + \sigma x + u_k | \mathcal{I}_{ki}] \quad (2.5)$$

We focus on the case in which the size of two sectors is equal, *i.e.*, $n_1 = n_2$.⁷ In this case:

$$u_1 = -u_2 = u \quad (2.6)$$

Therefore, u summarizes the exogenous unknown aggregate state variables that matter for firms' optimization problem. The aggregate state u is normally distributed according to:

$$u \sim N(0, \sigma_u^2) \quad (2.7)$$

Each firm i in sector k receives a signal $s_{k,i}$ about the unobserved state u :

$$s_{k,i} = u + e_{k,i} \quad \text{with} \quad e_{k,i} \sim N(0, \sigma_e^2) \quad (2.8)$$

Each firm draws a signal with the same precision, but realizations are different, and therefore, beliefs are dispersed across firms. Firms apply Bayes' theorem to form beliefs

⁶In particular, $y^N \equiv \frac{1}{\sigma}(n_1 a_1 + n_2 a_2)$.

⁷When the size of the two sectors is not equal, $u_1 + \frac{n_1}{n_2} u_2 = 0$ and our following results are robust to it.

about the unobserved state:

$$E(u|\mathcal{I}_{ki}) = Ks_{k,i}, \quad (2.9)$$

with $K = \frac{\sigma_u^2}{\sigma_e^2 + \sigma_u^2}$. Firms have the same belief updating rule governed by the signal-noise ratio. However, beliefs are dispersed due to the different signals that firms receive.

The Central Bank The central bank conducts a price index stabilization policy. At the beginning of the world, before the realization of shocks, the central bank is perfectly informed about the structure of the economy. The central bank chooses the price index that she commits to stabilize when shocks hit the economy. That is, ex-ante the central chooses the weight, ω , on sector 1's price, such that ex-post the price index is constant.

$$\omega p_1 + (1 - \omega)p_2 = 0, \quad (2.10)$$

In a special case, when $\omega_k = n_k$, the price index that the central bank commits to stabilize coincides with the CPI.

We will analyze two types of monetary policy. The first is that the central bank commits to stabilize the CPI. In the second scenario, the central chooses the price index optimally. In particular, optimal price index stabilization policy minimizes welfare loss function subject to the equilibrium conditions (2.5) with the definition of aggregate price p , (2.10) and firms' beliefs updating rule (2.9). The welfare loss function will be derived later.

The Timing of the Model The model consists of four stages. In stage one, the central bank chooses the inflation index that she commits to stabilize. In stage two, nature draws

a shock u , and each firm i in sector k receives a private signal $s_{k,i}$ about u . In stage three, each firm i forms a belief and decides a price-setting plan. In the last stage, the representative household observes the state of the economy and makes the consumption and labor decision. Simultaneously, the goods and labor markets clear.

Model Solution The solution of the model is characterized by (2.5) with the definition of aggregate price p , (2.10) and the firms' beliefs updating rule (2.9).

Solving those equations, we obtain the following aggregate allocations:

$$p_1^* = 2(1 - \omega)Ku, \quad (2.11)$$

$$p_2^* = -2\omega Ku, \quad (2.12)$$

$$x = -(1 - K + \theta K)(2\omega - 1)u \quad (2.13)$$

where x denotes the output gap.

2.2 Welfare analysis

Our objective is to evaluate the consequence of a change in nominal rigidity θ_k in terms of welfare. To this end, we derive the welfare loss function as the second order approximation of the household's utility function:

$$L = \sum_{k=1}^K \left(\epsilon n_k \text{var}_i \{ p_{k,i} \} \right) + \sigma x^2 + \eta \sum_{k=1}^K n_k \tilde{p}_{R,k}^2 \quad (2.14)$$

By combining the welfare loss function with the solution of the model, we show that the unconditional welfare losses can be rewritten as:

$$\begin{aligned}
E(L) = \epsilon \sum_{i=1}^2 & \left\{ \underbrace{2(1-\theta_i)\theta_i(1-\omega)^2 K^2 \sigma_u^2}_{\text{Calvo}} + \underbrace{2(1-\theta_i)(1-\omega)^2 K^2 \sigma_e^2}_{\text{Dispersed belief}} \right\} \\
& \underbrace{\hspace{10em}}_{\text{Price dispersions}} \\
& + \underbrace{\sigma \left\{ (1-K+\theta K)^2 (2\omega-1)^2 \sigma_u^2 \right\}}_{\text{Output gap volatility}} + \underbrace{2\eta(1-K+\theta K)^2 \sigma_u^2}_{\text{Relative price gap across sectors}} \quad (2.15)
\end{aligned}$$

The welfare loss function consists of three components: i) price dispersions within a sector, ii) output gap volatility and iii) relative price gap across sectors (or relative output gap). Such dispersions and volatilities reflect the degree of inefficiency of the economy because goods matter for households' utility symmetrically and production technologies are identical.

The novelty, in a dispersed belief model, as compared to the perfect information case, is that two channels drive price dispersions. The first is the standard *Calvo channel* that leads to price dispersions across the group of firms who can reset prices with those who cannot. The second is the *dispersed belief channel* that generates price dispersions *within* the group of price resetting firms. Firms form different beliefs about the state of the economy. Therefore, in contrast to a standard model with perfect information, price dispersion emerges among those firms that can reset prices.

Symmetric Reduction in Nominal Frictions We are now ready to discuss the welfare implication of an increase in price flexibility. We begin with the analysis of a special case in which the two sectors are symmetric with the same degree of nominal rigidity i.e., $\theta_1 = \theta_2 = \theta$, but are subject asymmetric shocks. We study how a change in the degree of nominal rigidity in the entire economy, i.e., θ , affects welfare. We will derive an analytical

solution for the welfare losses as a function of the Calvo parameter θ under the assumption that the central bank fully stabilizes the CPI. It is worth noting that our results do not depend on this assumption. As long as zero welfare losses are not achievable, which is always the case in a multi-sector model due to asymmetric shocks (Woodford, 2011), our results will always survive. Moreover, within the family of full price index stabilization policy, the full stabilization of the CPI is the optimal one in a multi-sector model with symmetric sectoral characteristics.

We begin with the discussion of the case in which there is no information imperfection, i.e., $\sigma_e = 0$.

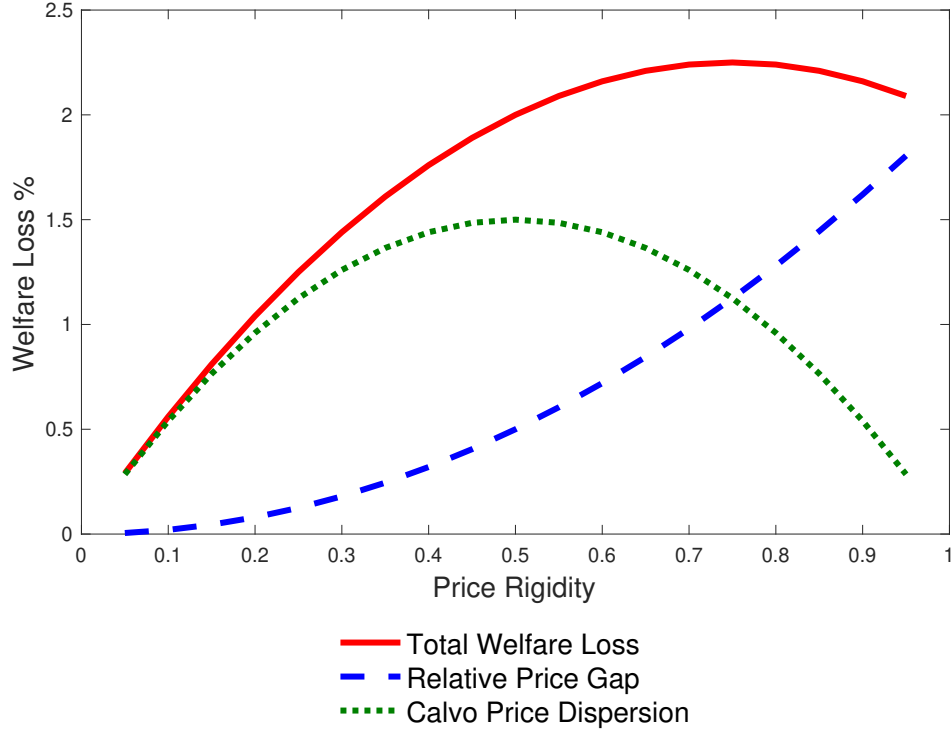
Proposition 1. *Consider a two-sector model with symmetric sectoral characteristics. Under CPI index stabilization policy, and with perfect information, when $\eta \geq \frac{1}{4}\epsilon$, the welfare losses are increasing in $\theta \in (0,1)$; when $\eta < \frac{1}{4}\epsilon$, the welfare loss function is concave in θ , and it attains the maximum at $\theta^{max} = \frac{\epsilon}{2\epsilon - 4\eta}$.*

Note that η determines the relative importance of the relative price gap in the welfare loss function. The welfare losses from the relative price gap are monotonically increasing in θ . The higher is the nominal rigidity (a higher θ), the bigger is the gap between the relative price (across sectors) and the desired one, hence bigger welfare losses. Therefore, if η is relatively large $\eta \geq \frac{1}{4}\epsilon$, the relative price gap component is dominating in the welfare loss function, which explains the first half of the Proposition (1). The standard calibration in the literature is to set η equal to 1. Although researchers disagree on the values for ϵ , they are normally bigger or equal to 6. Hence, the condition $\eta \geq \frac{1}{4}\epsilon$ is violated and we proceed to the discussion of the second half of the proposition.

Now consider the Calvo price dispersion component and the output gap volatility component. In general, the output gap volatility component is weakly increasing in the

degree of price rigidity θ . However, in the symmetric case that we consider here, the stabilization of the CPI is equivalent to the full stabilization of the output gap, as it can be inferred from equation (2.5). Therefore, the output gap is always equal to zero across different values of θ . Price dispersion originating from the Calvo pricing friction is a hump-shaped function of nominal rigidity. To understand this, consider the two extreme cases: one with full price flexibility ($\theta = 0$) and one with full price rigidity ($\theta = 1$). In both cases, firms either all reset the same optimal price or have to keep the initial price (steady-state price) that is the same across all firms. Hence, there is no price dispersion in those two extreme cases. When $\theta \in (0,1)$, the dispersion in prices arises among the group of firms who can reset prices with those who cannot change their prices. Given that the loss function is well-behaved, it must be a concave function in $\theta \in (0,1)$. Moreover, the welfare losses peak at the point when $\theta^{max} = \frac{\epsilon}{2\epsilon-4\eta}$. Naturally, θ^{max} depends on ϵ and η as those parameters capture the relative importance of price dispersion within a sector and relative price gap across sectors for welfare. This hump shape loss function thus suggests that if a decrease in price rigidity (θ) is welfare-improving if it takes place in the parameter region $\theta < \theta^{max}$. To give a sense of how big this parameter space is, let's take the popular calibration described above: $\epsilon = 6$, $\eta = 1$. θ^{max} is equal to 0.75, a value that coincides with the popular calibration of the Calvo parameter in the literature and it is of a similar magnitude with the empirical estimate provided by Nakamura and Steinsson (2008). Therefore, a model with full information predicts that given the current nominal rigidity in the economy, an increase in price flexibility is welfare-improving. Figure (1) provides a visual representation of those results discussed above by plotting the components of the welfare losses as a function of price rigidity using the calibration described above.

Figure 1: Decomposition of the Welfare Losses: Perfect Information



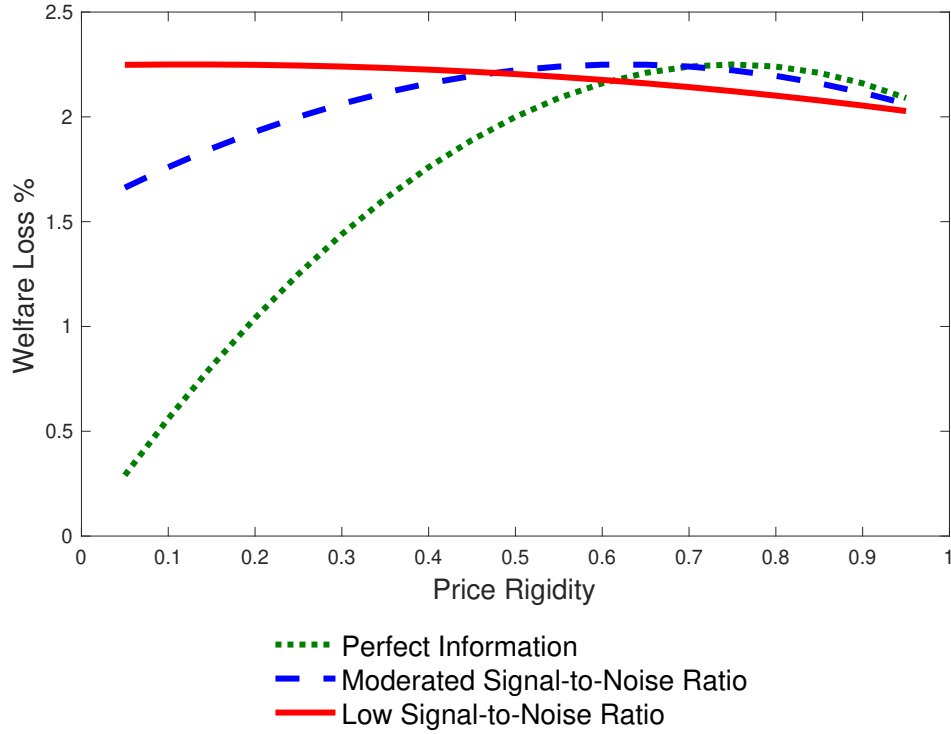
Note: This figure plots each component of welfare losses and total welfare losses as a function of price rigidity for the case with perfect information.

In the following discussion, we allow for information frictions and dispersed beliefs.

Proposition 2. *Consider a two-sector model with symmetric sectoral characteristics. Under CPI stabilization policy, when $\eta \geq \frac{1}{4}\epsilon$, the welfare losses are increasing in $\theta \in (0,1)$; when $\eta < \frac{1}{4}\epsilon$, the welfare loss function is concave in θ , and it attains the maximum at $\theta^{max} = \frac{\epsilon}{2\epsilon-4\eta} - \frac{\epsilon-4\eta}{2\epsilon-4\eta} \frac{\sigma_e^2}{\sigma_u^2}$, and θ^{max} is increasing in the signal-to-noise ratio $\frac{\sigma_a}{\sigma_e}$. When $\frac{\sigma_e^2}{\sigma_u^2} > \frac{\epsilon}{\epsilon-4\eta}$, the welfare losses are decreasing in $\theta \in (0,1)$.*

Figure (2) demonstrates proposition 2 visually. It plots the welfare losses as a function of price rigidity under different scenarios. The dotted green line presents the perfect information case (the same as the solid red line in Figure 1). The dashed blue line plots

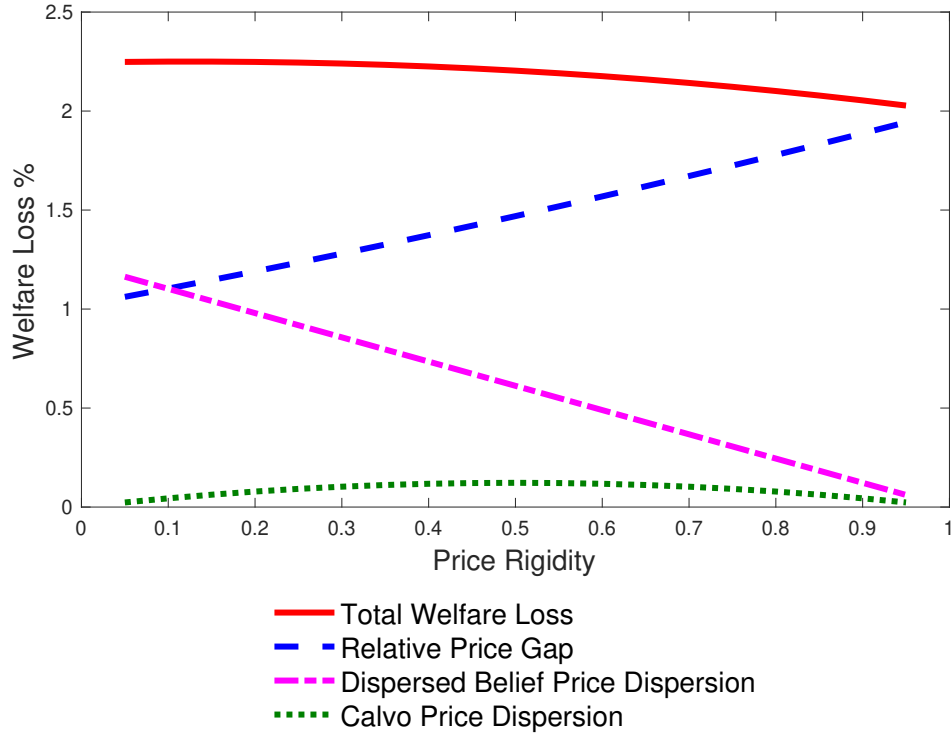
Figure 2: Welfare Losses under Different Level of Information Frictions



the prediction of the model with dispersed beliefs and moderated signal-to-noise ratio, i.e., moderated degree of information friction. The solid red line shows the result with a low level of the signal-to-noise ratio. If the degree of information friction is sufficiently strong, in contrast to the case with perfect information (solid black line), the paradox of price flexibility arises — reducing price rigidity is no longer welfare-improving!

The above result is entirely driven by the dispersed beliefs channel. With information imperfection and dispersed beliefs, dispersion in prices arises within the group of firms who can reset prices. The dispersed belief component of the welfare losses is strictly increasing in the price flexibility ($1 - \theta$). This is verified in Figure (3), which plots each component of the welfare losses as a function of price rigidity for the case with dispersed beliefs. The dash-dotted pink line is strictly decreasing in price rigidity.

Figure 3: Welfare Losses Decomposition: Information Frictions

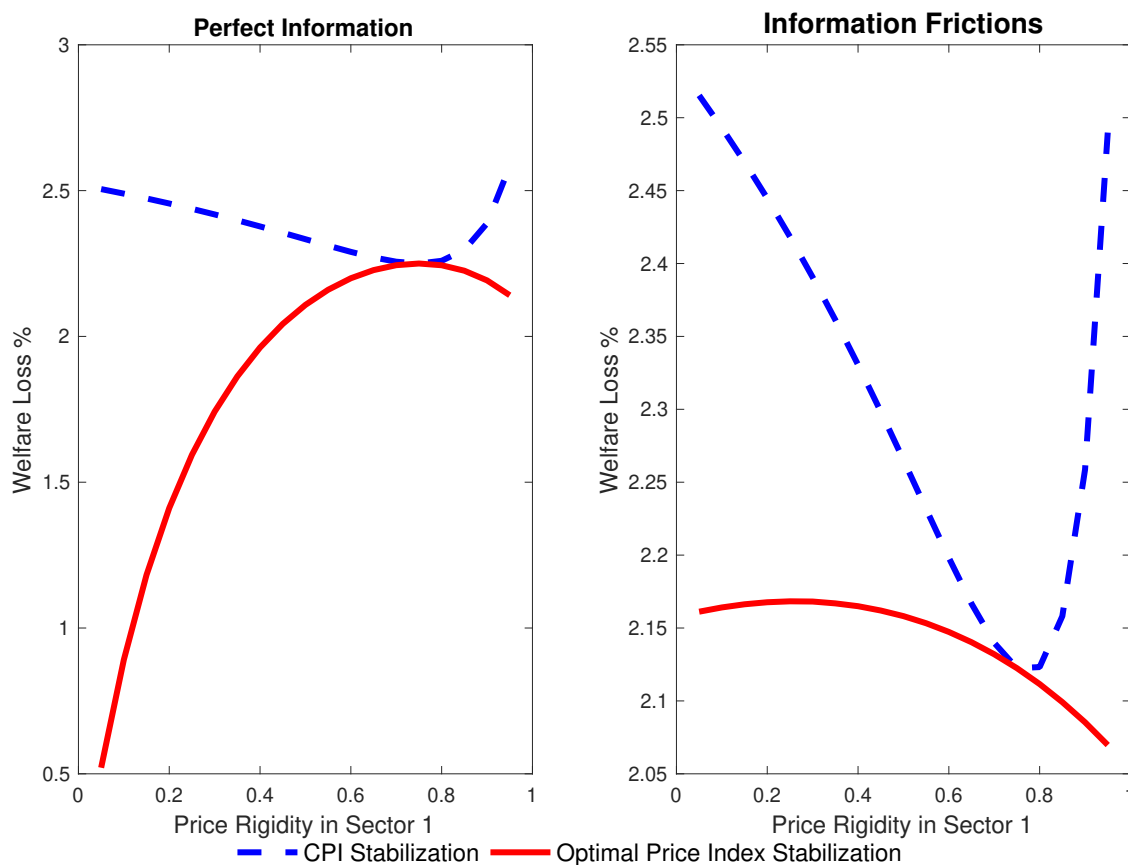


Asymmetric Reduction in Nominal Frictions So far, we have analyzed the consequence of an economy-wide reduction in θ . We now turn to the scenario that only one sector benefits from the technology that facilitate price adjustment, and therefore price flexibility increases in that sector. The remaining sectoral characteristics such as the sector size and the volatility of shocks are assumed to be the same.

Figure 4 shows how the welfare losses change when the degree of nominal rigidity in sector 1 varies. The dashed blue line in the left panel plots the result for the model with perfect information and with a central bank that commits to stabilize the CPI. As one can see, a decrease in θ_1 leads to higher welfare losses. To further investigate which component of the welfare loss function drives the result, we plot welfare decomposition under different calibration of θ_1 in Figure 8. Again, the dashed blue line corresponds to

the case where the central bank stabilize the CPI. Figure 8 shows that the main driver of the paradox is the price dispersion in sector 2: it increases substantially as prices in sector 1 becomes more flexible. The intuition is the following. Consider a positive productive shock that occurs in sector 1, output in this sector increases although not as much as the rise in its natural level due to nominal rigidity. This generates an increase in aggregate demand, which results in a positive output gap in sector 2 hence price in the second sector rises. Now if the nominal rigidity in sector 1 were lower, everything else equal, the initial increase in sector 1's output would be higher that would ultimately result into an even higher price in Sector 2. Therefore, a bigger price dispersion arises in sector 2.

Figure 4: Welfare Losses and Nominal Rigidity in Sector 1



The dashed blue line in the right panel of Figure 4 shows that the paradox of price flexibility holds for a model with information frictions under the CPI stabilization policy. However, a detailed inspection shows that the main source of the paradox is the price dispersion originated from dispersed beliefs. as it is evidenced by dashed blues in Figure 9.

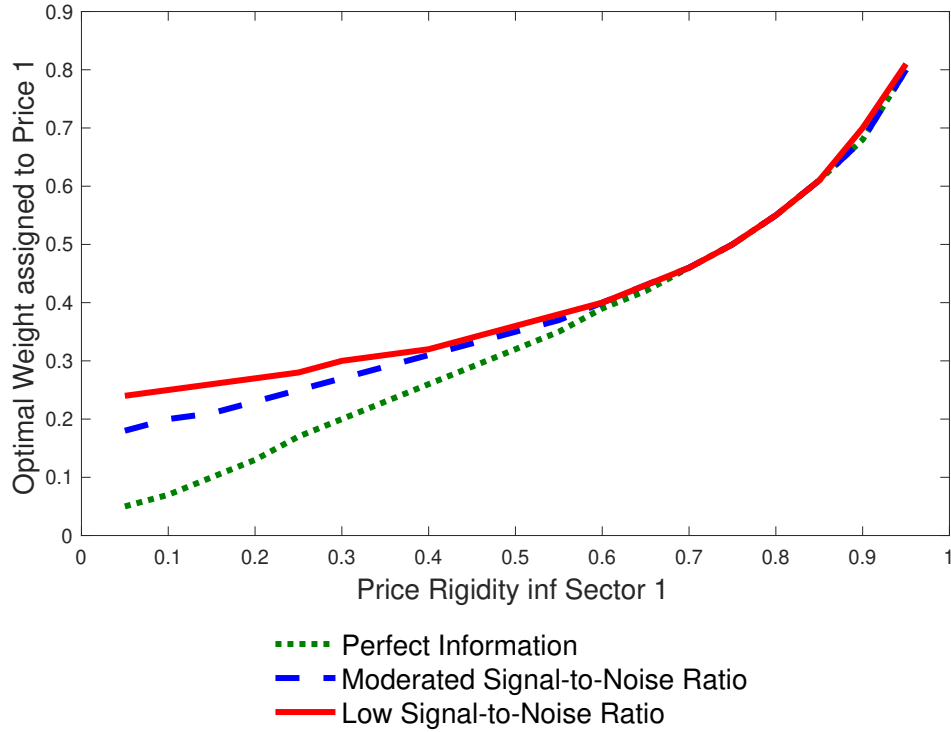
2.3 Optimal Inflation Index Stabilization Policy

We have looked at the welfare consequence of a reduction in θ_1 , holding everything else constant including the CPI stabilization policy. Clearly, as it is hinted by the substantial increase of price dispersion in sector 2, such a policy is suboptimal. We now turn to the discussion of the optimal inflation index stabilization policy and to check whether an optimal policy can resolve the paradox.

The solid red lines in Figure 4 plots the result of our thought experiment: how welfare losses vary with price rigidity in sector 1, conditional on a central bank that conducts the optimal price index stabilization policy. Under such a policy, for each value of θ_1 , the central bank recalculates the optimal price index—the one associated with the lowest welfare losses. Interestingly, the optimal inflation index stabilization policy resolves the paradox of price flexibility in a model with perfect information, as it is shown in the left panel. Under the optimal policy, the central bank is aware of the risk of high volatility of price in sector 2 when price in sector 1 becomes more flexible. As a response, the central bank assigns a higher weight to p_2 in the price index that it aims to stabilize. The dotted green line in Figure 5 plots this result: the optimal weight assigned to p_1 (p_2) is strictly increasing (decreasing) in θ_1 , consistent with the prediction of Benigno (2004). The success of this policy is shown in Figure 8. Under the optimal price index stabilization policy (red

solid lines), the price dispersion in sector 2 is dampened as compared to the case with the stabilization of the CPI, at the cost of a higher price dispersion in sector 1.

Figure 5: Optimal Price Index



In contrast to the model with perfect information, in the presence of information frictions and dispersed beliefs, the paradox of price flexibility survives even under the optimal price index stabilization policy, as it is shown in the right panel of Figure 4. This is due to the emergence of price dispersions originated from dispersed beliefs, see Figure 9. The optimal price index stabilization policy is unable to attenuate the welfare losses resulting from belief dispersions. As discussed above, a reduction in price rigidity in sector 1 leads to a bigger price adjustment in sector 2, and therefore the central bank reacts to assign a higher weight on p_2 at the cost of a more volatile price in sector 1. However, the welfare cost of price volatility in sector 1 decreases as its nominal friction declines. This is

Benigno (2004)'s result. In contrast, in the presence of dispersed beliefs, price dispersion among price resetting firms arises. A more volatile price in sector 1 results into a larger price dispersions due to dispersed beliefs, which is costly in terms of welfare. Moreover, as the fraction of price resting firms increase, the dispersed beliefs price dispersion channel becomes more dominant. As a result, it offsets the Calvo channel highlighted in Benigno (2004). This is shown in Figure 5, in a model with information frictions, the optimal weight attached to p_1 decreases as θ_1 drops, however the decline is more moderated as compared to the case with perfect information.

This result hints on the importance of the consideration of information frictions in the design of optimal inflation index stabilization policy. To the best of our knowledge, this is not yet addressed in the literature. We postpone the discussion about the quantitative importance of a such consideration to the dynamic version of the model that we present in the next section.

Summing up To sum up what we have derived so far in a static model. First, in a symmetric two-sector model, an economy-wide increase in price flexibility is likely to be welfare-improving in the absence of information frictions. Second, with information frictions and dispersed beliefs, the paradox of price flexibility arises. Third, this paradox arises independent of information frictions if the reduction in nominal frictions is merely a sectoral phenomenon and the central bank stabilizes CPI. Fourth, the optimal price index stabilization policy is capable of resolving the paradox but only in a model with perfect information. Fifth, price dispersion from dispersed beliefs offsets the Calvo channel highlighted in Benigno (2004), and therefore affects the design of the optimal price index.

In order to evaluate the quantitative importance of those results, we extend our model

to a dynamic version.

3 Dynamic model

In this section, we present a dynamic multi-sector model with dispersed beliefs. The two-sector economy is populated by a continuum of $(0,1)$ of households, a fraction of n_k of monopolistic competitive firms in sector k for $k = 1, 2$, and a fiscal-monetary authority. A fraction of θ_k of firms (Calvo probability) in sector k are allowed to reset their prices in each period. Households consume the composite goods, buy a one-period risk-free government bond, supply labor to the sectoral competitive labor market, receive dividends (profits) from firms and pay taxes or receive transfers from the fiscal-monetary authority. Firms demand labor to produce and sell goods to households. The fiscal-monetary authority issues government bonds, collect lump-sum taxes or pay transfers to households, sets the nominal interest by following a simple rule and chooses the inflation target weight to maximize social welfare.

3.1 The economy

Households Households have complete information. The representative household chooses consumption, bond holding, and labor supply to maximize her lifetime utility:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \sum_{k=1}^K \frac{L_{k,t}^{1+\varphi}}{1+\varphi} \right)$$

where β is the discount factor, σ is the relative risk aversion and φ governs the elasticity of labor supply. The budget constraint of the household in period t reads as:

$$P_t C_t + Q_t B_{t+1} = B_t + \sum_{k=1}^K W_{kt} L_{kt} + \sum_{k=1}^K \Pi_{kt} + T_t \quad (3.1)$$

where P_t is the price level of the composite good, Q_t is the nominal one-period risk-free bond price, and W_{kt} is the sectoral nominal wage, Π_{kt} stands for sectoral profit, and T_t stands for lump-sum taxes/transfers. The composite good that enters into household's utility function is a Dixit-Stiglitz aggregator of the following form:

$$C_t = \left[\sum_{k=1}^K n_k^{1/\eta} C_{k,t}^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)},$$

where n_k is the size of the sector k and the weights n_k sum up to one and $C_{k,t}$ is sectoral output which is a Dixit-Stiglitz aggregator of the following form:

$$C_{k,t} = \left[n_k^{-1/\varepsilon} \int_0^{n_k} C_{ki,t}^{(\varepsilon-1)/\varepsilon} di \right]^{\varepsilon/(\varepsilon-1)}.$$

The bond holdings satisfy a no-Ponzi-scheme condition.

Solving the consumer's problem yields the following demand functions:

$$C_{k,t} = \left(\frac{P_{k,t}}{P_t} \right)^{-\eta} n_k C_t \quad C_{ki,t} = \left(\frac{P_{ki,t}}{P_{kt}} \right)^{-\varepsilon} \frac{1}{n_k} C_{kt} \quad (3.2)$$

where the implied aggregate price and sectoral prices level are:

$$P_t = \left[\sum_{k=1}^K n_k P_{k,t}^{1-\eta} \right]^{1/(1-\eta)} \quad P_{k,t} = \left[n_k^{-1} \int_0^{n_k} P_{ki,t}^{1-\varepsilon} di \right]^{1/(1-\varepsilon)}. \quad (3.3)$$

Firms There is a continuum of monopolistic competitive firms in each sector. Each firm i in a sector k is endowed with a common linear production function:

$$Y_{ki,t} = e^{a_{k,t}} L_{ki,t},$$

where the log productivity $a_{k,t}$ is homogenous within a sector and it evolves according to an autoregressive process:

$$a_{k,t} = \rho_k a_{k,t-1} + \epsilon_{k,t}$$

where the sectoral technology shock follows a normal distribution with mean zero and standard error σ_{a_k} .⁸

Firms are subject to nominal rigidity à la Calvo (1983): each firm in a sector k may reset its price with probability $(1 - \theta_k)$. Hence, the log level price at sector k , p_{kt} , evolves as the following:

$$p_{kt} = \theta_k p_{kt-1} + (1 - \theta_k) p_{kt}^*,$$

where p_{kt}^* is the average of the optimal price that a re-optimizing firm at sector k would set to, the latter is the solution to the following problem:

$$\max_{p_{ki,t}^*} \sum_{h=0}^{\infty} \theta_k^h E_t \left\{ Q_{t,t+h} (P_{ki,t}^* Y_{ki,t+h|t} - \Psi_{ki,t+h}(Y_{ki,t+h|t})) \right\}$$

subject to the demand schedule specified in (3.2). $Q_{t,t+h} \equiv \beta^h (C_{t+h}/C_t)^{-\sigma} (P_t/P_{t+h})$ de-

⁸We abstract from firm level productivity shocks. In the presence of idiosyncratic shocks, the optimal target inflation rate will not be equal to zero: see [Blanco \(2016\)](#) and [Adam and Weber \(2019\)](#) for a detailed discussion. However, incorporating idiosyncratic shocks will not affect the main message of our paper: core inflation is not the optimal choice of inflation index.

notes the stochastic discount factor, $\Psi_{ki,t+h}$ is the nominal cost function and $Y_{ki,t+h|t}$ is the output for a firm i in sector k that last reset its price in period t .

The optimality condition implied by the firm's problem is:

$$\sum_{h=0}^{\infty} \theta_k^h E_t \left\{ Q_{t,t+h} Y_{ki,t+h|t} (P_{ki,t}^* - \frac{\varepsilon}{\varepsilon-1} \Psi'_{ki,t+h}(Y_{ki,t+h|t})) \right\} = 0.$$

The monetary policy The central bank has a unique mandate of inflation stabilization. in particular, it commits to stabilize the following inflation index:

$$\omega_1 \pi_1 + (1 - \omega_1) \pi_2 = 0.$$

Similar to the static model, we analyze two types of policies: i) a CPI stabilization policy, and ii) the central bank chooses ω_1 optimally to minimize the welfare losses.

Firm's information set Both firms and households are rational and know the structure of the economy. However, firms can not observe the true sectoral TFP shocks. Instead, firms receive signals about sectoral TFP shocks. Specifically, a firm i in sector l receives a signal $s_{k,l,t}(i)$ about sectoral shock a_{kt} for all k in period t :

$$s_{k,l,t}(i) = a_{kt} + e_{k,l,t}(i) \quad \text{with} \quad e_{k,l,t}(i) \sim N(0, \sigma_{e_k}^2) \quad (3.4)$$

where $e_{k,l,t}(i)$ is a white noise. The precision of signals are common across firms in the entire economy. That is, we abstract from heterogeneity in the degree of information frictions among firms. Imperfect signals capture the idea that the firms might not pay full

attention to economic indicators. The information set of a firm i in sector l is:

$$\mathcal{I}_{l,t}(i) \equiv \{s_{k,l,\tau}(i), \hat{y}_{R,kt-1}, P_{ki,t} : \forall k, \forall l, \forall \tau \leq t\}.$$

Note that we have made the following simplifying assumptions that are common in the literature.⁹ First, firms are allowed to observe the lagged state variable, i.e., $\hat{y}_{R,kt-1}$ the lagged relative output. Otherwise, firms would need to form high-order beliefs about $\hat{y}_{R,kt-1}$ that increases the number of state variables in the model exponentially. Second, $s_{k,l,t}(i)$ summarizes all remaining information that firms possess and conceptually we exclude the possibility that firms observe output and labor perfectly and simultaneously. That is, they cannot back up the underlining unobserved productivity shocks perfectly.

It is worth mentioning that a firm's action, $P_{ki,t}$, is a redundant piece of information as its action is based on its posterior belief. Therefore, it does not reflect any further information.

3.2 Linearized Equilibrium Conditions

We log-linearize the household's and firm's problem around the perfect information deterministic steady state and obtain the dynamic IS equation and sectoral New Keynesian Phillips curve (or NKPC for short). Throughout this paper, a variable with tilde denotes this variable in deviation from its natural level. And a variable with hat denotes this variable in deviation from its steady-state. The imperfect-common-knowledge sectoral NKPC are obtained as the following:

$$\pi_{kt} = (1 - \theta_k)(1 - \beta\theta_k) \sum_{j=1}^{\infty} (1 - \theta)^{j-1} (\hat{mc}_{kt|t}^{(j)} - \hat{p}_{R,kt|t}^{(j)}) + \beta\theta_k \sum_{j=1}^{\infty} (1 - \theta_k)^{j-1} \hat{\pi}_{kt+1|t}^{(j)} \quad (3.5)$$

⁹See e.g., [Nimark \(2008\)](#) and [Melosi \(2016\)](#).

where

$$\hat{m}c_{kt|t}^{(j)} = \sigma(y_t - y^N)^{(j)} + \varphi(y_{kt|t} - y_{kt|t}^N)^{(j)} + \eta^{-1}(y_{t|t}^N - y_{kt|t}^N)^{(j)}. \quad (3.6)$$

Regarding notations, for a variable z_t the following notation for high-order expectations are used:

$$\begin{aligned} z_{t|t}^{(0)} &\equiv z_t \\ z_{t|t}^{(1)} &\equiv \int \int E(z_t | \mathcal{I}_{l,t}(i)) di dl \\ z_{t|t}^{(2)} &\equiv \int \int E(z_{t|t}^{(1)} | \mathcal{I}_{l,t}(i)) di dl \\ z_{t|t}^{(j)} &\equiv \int \int E(z_{t|t}^{(j-1)} | \mathcal{I}_{l,t}(i)) di dl. \end{aligned}$$

From the goods demand equation, the relative price relates to the relative output according to:

$$\hat{p}_{R,kt|t}^{(j)} = -\eta^{-1}(y_{kt|t} - y_{t|t})^{(j)} \quad (3.7)$$

so that the sectoral NKPC can be rewritten as:

$$\begin{aligned} \pi_{kt} = (1 - \theta_k)(1 - \beta\theta_k)\Theta \sum_{j=1}^{\infty} (1 - \theta_k)^{j-1} \Big[(\sigma + \varphi) \tilde{y}_{t|t}^{(j)} + \\ (\eta^{-1} + \varphi) (\hat{y}_{R,kt|t}^{(j)} - \Phi(a_{kt|t}^{(j)} - \sum_{k=1}^K n_k a_{kt|t}^{(j)})) \Big] + \beta\theta_k \sum_{j=1}^{\infty} (1 - \theta_k)^{j-1} \hat{\pi}_{kt+1|t}^{(j)}. \end{aligned} \quad (3.8)$$

where $\Phi \equiv \frac{1+\varphi}{\eta^{-1}+\varphi}$, and we have used the fact that $\hat{y}_{R,kt|t}^{N,(j)} = \Phi(a_{kt|t}^{(j)} - \sum_{k=1}^K n_k a_{kt|t}^{(j)})$. In (3.8), the high-order expectations are weighted by $(1 - \theta_k)^{j-1}$ and the impact of expectations is decreasing as the order of expectation increases. The smaller the Calvo parameter θ_k , the more important high-order expectations are.

The law of motion for $\hat{y}_{R,kt}$ is:

$$\hat{y}_{R,kt} = -\eta(\pi_{kt} - \sum_{k=1}^K n_k \pi_{kt}) + \hat{y}_{R,kt-1}, \quad (3.9)$$

where we have used the definition of π_{kt} , π_t , and equation (3.7) for $j = 0$. Therefor, one can easily see that $\hat{y}_{R,kt-1}$ is a state variable in period t .

The Dynamic IS equation is obtained by log-linearizing the Euler equation:

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \frac{1}{\sigma} (i_t - E_t \sum_{k=1}^K n_k \pi_{kt+1} - r_t^N) \quad (3.10)$$

where $r_t^N = \rho + \sigma \psi^a \sum_{k=1}^K n_k E \Delta a_{kt+1}$ with $\psi^a = \frac{1+\varphi}{\sigma+\varphi}$.

Finally recall the monetary authority's policy rule that completes the description of the economy:

$$\omega_1 \pi_1 + (1 - \omega_1) \pi_2 = 0. \quad (3.11)$$

3.3 Solution

We follow the method described in [Nimark \(2008\)](#) and [Melosi \(2016\)](#) to solve the two-sector model with imperfect-common-knowledge. We leave the detailed description of the algorithm to the Appendix [A.4](#). When the model is solved, the policy function for endogenous variables $s_t \equiv [\pi_{kt} \tilde{y}_t \hat{y}_{R,kt}]'$ reads as:

$$s_t = v_0 X_{t|t}^{(0:J)} \quad (3.12)$$

where the state vector $X_{t|t}^{(0:J)} \equiv [a_{1,t|t}^{(s)}, a_{2,t|t}^{(s)}, \hat{y}_{R,1t-1} : 0 \leq s \leq J]'$ contains the hierarchy of average high-order expectations about the exogenous state variables $X_t \equiv [a_{1t} \ a_{2t}]'$. As

a standard way to tackle the infinite dimension of the state vector, we truncated it up to order J to keep the dimension of state vector finite. The state equation can be summarized as:

$$X_{t|t}^{(0:J)} = MX_{t-1|t-1}^{(0:J)} + N\mathbf{f}\mathbf{f}_t. \quad (3.13)$$

To keep notation simple, denote $\mathbf{X}_t \equiv X_{t|t}^{(0:J)}$ and denote $Z_{lt}(i) \equiv [s_{1,l\tau}(i) \ s_{2,l\tau}(i) \ y_{R,1t-1}]'$ as signals received by firm i in sector l in period t . Then the observation equation becomes:

$$Z_{lt}(i) = L\mathbf{X}_t + Q\mathbf{e}_{lt}(i). \quad (3.14)$$

Equation (3.13) and (3.14) consist of a linear state space, which calls for the Kalman filter to estimate the unobserved state vector that contains high-order expectations. Simultaneously, the Kalman filter also determines the law of motion of the high-order beliefs, that is, determines the matrices M and N in (3.14). Hence the problem of solving the model boils down to finding a fixed point over the parameters v_0 that characterizes the policy function (3.12) and M and N that governs the law of motion of the state vector (3.13).

Note that so far we have solved the economy for any given the inflation index that the central bank targets in the Taylor rule.

3.4 Welfare loss function and policy

Following Rotemberg and Woodford (1997, 1999) and Woodford (2002), the second order approximation of the representative consumer's period welfare loss expressed in con-

sumption equivalent variation (CEV) is:

$$L = \sum_{k=1}^K \frac{\varepsilon n_k}{\Theta} \text{var}_i\{p_{ki,t}\} + (\sigma + \varphi) \text{var}(\tilde{y}_t) + \sum_{k=1}^K (\eta^{-1} + \varphi) n_k \text{var}(\tilde{y}_{R,kt}). \quad (3.15)$$

Similar to a multi-sector model with complete information, the welfare loss function consists of i) price dispersions; ii) the volatility of aggregate output gap; and iii) the volatility of relative output gap across sectors. The novelty in the presence of information frictions is that the price dispersions can be further decomposed into:

$$\text{var}_i\{p_{ki,t}\} = \frac{1}{1 - \beta\theta_k} [(1 - \theta_k) \int_i (p_{ki,t}^* - p_{k,t}^*)^2 di + \frac{\theta_k}{1 - \theta_k} \text{var}(\pi_{k,t})]. \quad (3.16)$$

In contrast to the case with complete information, dispersion across price resetting firms $\int_i (p_{ki,t}^* - p_{k,t}^*)^2 di$ arises because firms have different assessments about the state of the economy. The detailed derivation for the welfare loss function can be found in the Appendix.

Central bank's optimization problem As discussed earlier, under an optimal price index stabilization policy, the benevolent monetary authority chooses inflation target weight $\{\omega_k\}$ to minimize the unconditional welfare losses. The problem for the monetary authority can be formalized as follows:

$$\min_{\{\omega_k\}} \sum_{k=1}^K \frac{\varepsilon n_k}{\Theta} \frac{1}{1 - \beta\theta_k} E \left(\left[(1 - \theta_k) \int_i (p_{ki,t}^* - p_{k,t}^*)^2 di + \frac{\theta_k}{1 - \theta_k} \text{var}(\pi_k) \right] \right. \\ \left. + (\sigma + \varphi) \text{var}(\tilde{y}_t) + \sum_{k=1}^K (\eta^{-1} + \varphi) n_k \text{var}(\tilde{y}_{R,kt}) \right)$$

subject to equilibrium conditions discussed in the previous section.

This welfare loss function nests the standard welfare loss function with perfect infor-

mation as a special case. Without dispersed beliefs, each firm i in a sector k would set the same price, i.e., $\int_i (p_{ki,t}^* - p_{k,t}^*)^2 di = 0$. In the absence of information frictions, the inflation of a flexible price sector ($\theta_k = 0$) does not enter in the welfare loss function. In other words, inflation in this sector is costless in terms of welfare. This is the key result derived by [Aoki \(2001\)](#), and [Woodford \(2011\)](#) provides a textbook treatment.

However, in the presence of dispersed beliefs, the flexible price sector's inflation generates a welfare loss as it creates price dispersion across re-optimizing firms. Targeting the core inflation is no longer optimal.

4 Results

Calibration The calibrations of parameters are listed in Table (1). The utility function is log-linear in consumption and linear in labor. To highlight the new channel introduced by information frictions, we consider the two-sector economy in which sectors are identical in the following characteristics: the size of the sector, the process for productivity shocks and elasticity of substitution. However, sectors differ in the degree of nominal rigidity. In particular, we allow nominal rigidity in sector 1 to vary and fix the nominal rigidity in sector 2 such that $\theta_2 = 0.75$. Unless otherwise specified, the signal-to-noise ratio is calibrated to 1 to match the empirical Kalman gain estimated by [Coibion and Gorodnichenko \(2012, 2015\)](#).

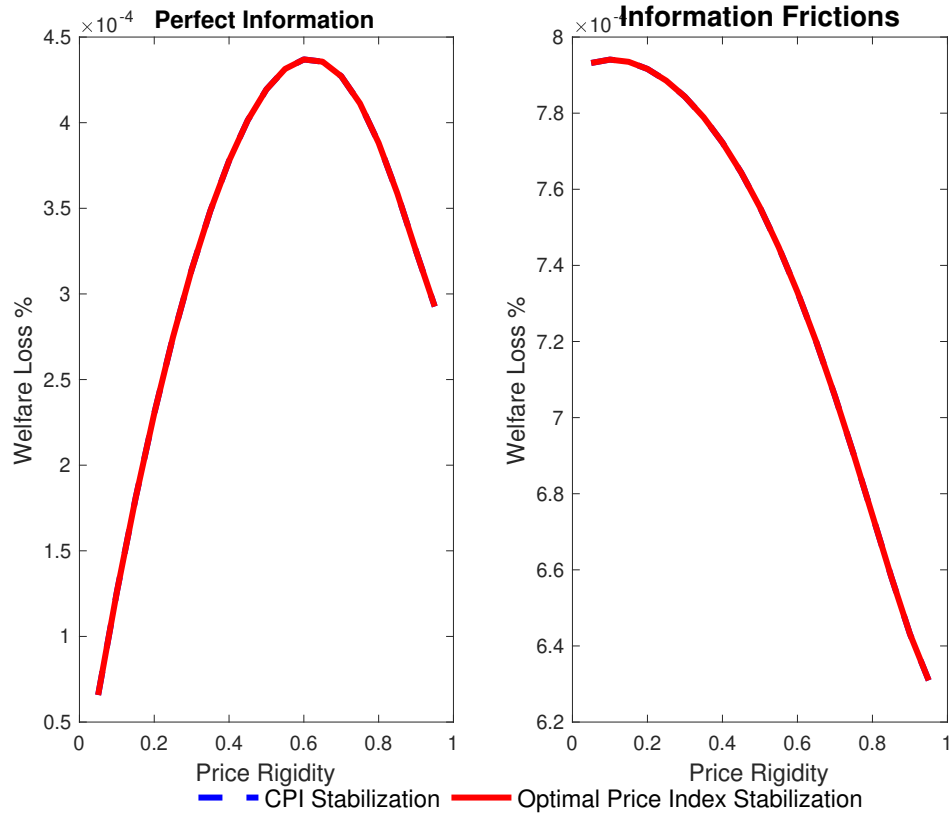
Symmetric Reduction in Nominal Frictions Figure 6 plots the welfare losses against an economy-wide change in nominal rigidity, both for a model with perfect information (the left panel) and with information frictions (the right panel). Note that, we conducted this thought experiment both under the CPI stabilization and an optimal price index stabiliza-

Utility function	$\sigma = 1, \varphi = 0$
Discount factor	$\beta = 0.99$
Size of sectors	$n_1 = 0.5, n_2 = 0.5$
Exogenous shocks	$\rho_1 = \rho_2 = 0.6, \sigma_{a_1}^2 = \sigma_{a_2}^2 = \sigma_a = (0.02)^2$
Signal-to-Noise ratio	$\sigma_a / \sigma_e = 1$
Nominal rigidity	$\theta_2 = 0.75$
Elasticity of substitution	$\eta = 1, \varepsilon_1 = \varepsilon_2 = 6$

Table 1: Calibration

tion policy. Since sectors are symmetric, the optimal price index coincides with the CPI and therefore the two curves overlap. Without information frictions, given the current level of nominal rigidity in the U.S., i.e., $\theta = 0.75$ that corresponds to an average price duration of four quarters, it would be welfare-deteriorating if the price is more flexible. However, a significant reduction in nominal friction, that lowers the average price duration to two quarters (θ smaller than 0.5), would improve social welfare. In contrast, the presence of information frictions aggravates the paradox substantially: an economy-wide improvement in nominal frictions is always welfare-deteriorating! Furthermore, the central bank, by setting the inflation index optimally, cannot resolve this paradox.

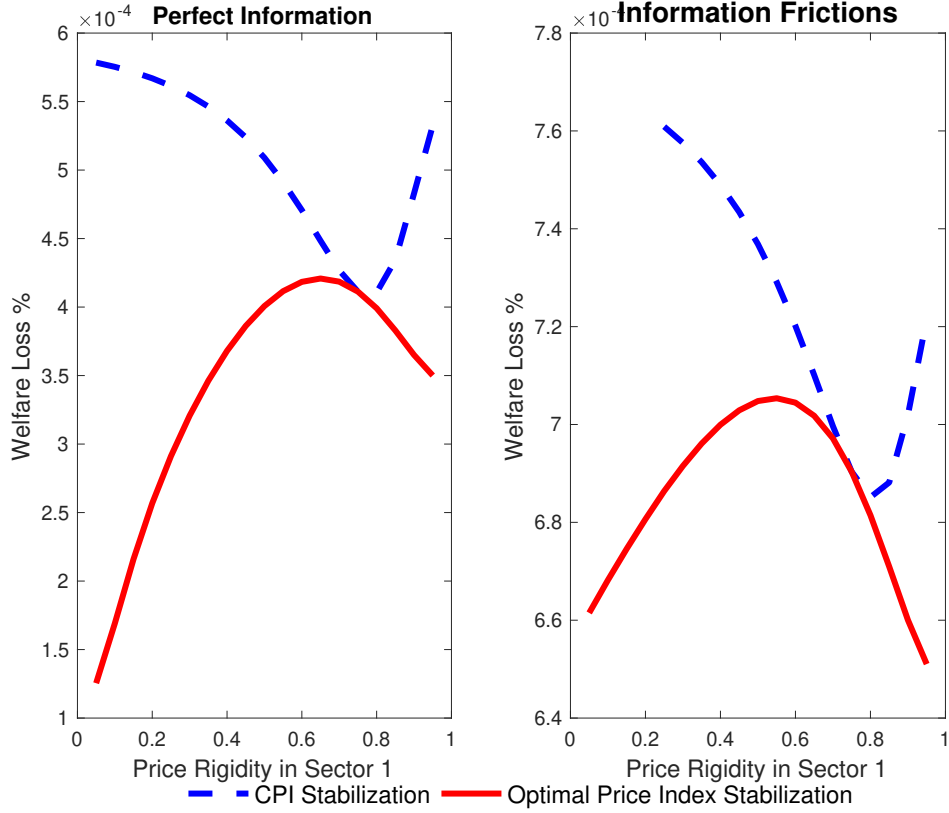
Figure 6: Welfare Losses: Economy-wide changes in Nominal Frictions



Asymmetric Reduction in Nominal Frictions We now consider the thought experiment when only nominal rigidity in sector 1 varies. Figure 7 plots the result, both for a model with perfect information (the left panel) and with information frictions (the right panel). As explained above in the static model, the paradox of price flexibility arises even in a model with perfect information if the inflation index that the central bank stabilizes is a sub-optimal one (CPI). An increased price flexibility in sector 1 brings about a more volatile price in sector 2, which is very costly if the central bank does not adjust its policy properly. The optimal inflation index stabilization policy mitigates the paradox. In this case, for it to be welfare-improving, the cut in nominal rigidity needs to surpass more than 1.5 quarters of reduction in average price duration if the information were perfect,

and more than 2.5 quarters in the presence of information frictions.

Figure 7: Welfare Losses: asymmetric changes in Nominal Frictions



5 Conclusion

We have derived the following results, based on a multi-sector NK model with dispersed beliefs. First, in a symmetric two-sector model, an economy-wide increase in price flexibility is likely to be welfare-improving in the absence of information frictions. Second, with information frictions and dispersed beliefs, the paradox of price flexibility arises. Third, this paradox arises independent of information frictions if the reduction in nominal frictions is merely a sectoral phenomenon and the central bank stabilizes CPI. Fourth, the optimal price index stabilization policy is capable of resolving the paradox but only in

a model with perfect information. Fifth, with price dispersion originated from dispersed beliefs offsets the Calvo channel highlighted in [Benigno \(2004\)](#), and therefore affects the design of the optimal price index.

Increased price flexibility might not be far away until it reaches our society, given the current trend of technological progress. Such progress that is beneficial from an individual firm's perspective might be harmful to social welfare. Of course, the first best solution to resolve the paradox of price flexibility is to reduce information frictions. However, this might be challenging. For example, if information imperfection arises from a human's limited ability of reasoning or rational inattention, as it is emphasized by [Sims \(2003\)](#) and [Mackowiak and Wiederholt \(2009\)](#), there is little hope to reduce such friction. Our analysis suggests the important role that the central bank has to play to mitigate or resolve the paradox of price flexibility when the reduced nominal frictions is a sectoral phenomenon.

Our results point to a market inefficiency that deserves a further investigation in future research. In our model, from an individual firm's perspective, it would be better off if the firm was allowed to adjust its price more frequently as they would be able to respond to shocks on a timely manner. Therefore, firms would pay for new pricing technologies. However, if all individual firms adopt the new technology to facilitate the price adjustment, the aggregate welfare losses may increase, as we discussed above. Thus, the constrained social planner allowing for the presence of dispersed information would not introduce the new technology.

References

- Adam, K. and H. Weber (2019). Optimal trend inflation. *American Economic Review* 109(2), 702–37.
- Amano, R. and S. Gnocchi (2017). Downward Nominal Wage Rigidity Meets the Zero Lower Bound. Technical report.
- Anand, R., E. S. Prasad, and B. Zhang (2015). What measure of inflation should a developing country central bank target? *Journal of Monetary Economics* 74(C), 102–116.
- Andrade, P., R. K. Crump, S. Eusepi, and E. Moench (2016). Fundamental disagreement. *Journal of Monetary Economics* 83, 106–128.
- Angeletos, G.-M., L. Iovino, and J. La’o (2016). Real rigidity, nominal rigidity, and the social value of information. *American Economic Review* 106(1), 200–227.
- Angeletos, G.-M. and L. Jennifer (2009). Incomplete information, higher-order beliefs and price inertia. *Journal of Monetary Economics* 56, S19–S37.
- Angeletos, G.-M. and J. La O (2013). Sentiments. *Econometrica* 81(2), 739–779.
- Angeletos, G.-M. and J. La’O (2011). Optimal monetary policy with informational frictions. Technical report, National Bureau of Economic Research.
- Angeletos, G.-M. and C. Lian (2018). Forward guidance without common knowledge. *American Economic Review* 108(9), 2477–2512.
- Aoki, K. (2001, August). Optimal Monetary Policy Responses to Relative-price Changes. *Journal of Monetary Economics* 48(1), 55–80.

- Basu, S. and P. D. Leo (2016, March). Should Central Banks Target Investment Prices? Boston College Working Papers in Economics 910, Boston College Department of Economics.
- Benigno, P. (2004, July). Optimal Monetary Policy in a Currency Area. *Journal of International Economics* 63(2), 293–320.
- Bhattarai, S., G. B. Eggertsson, and R. Schoenle (2018). Is increased price flexibility stabilizing? redux. *Journal of Monetary Economics* 100, 66 – 82.
- Billi, R. M., J. Galí, and U. CREI (2018). Gains from wage flexibility and the zero lower bound. Technical report.
- Blanco, A. (2016). Optimal inflation target in an economy with menu costs and zero lower bound. *Unpublished manuscript, University of Michigan*.
- Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics* 12(3), 383–398.
- Coibion, O. and Y. Gorodnichenko (2012). What can survey forecasts tell us about information rigidities? *Journal of Political Economy* 120(1), 116–159.
- Coibion, O. and Y. Gorodnichenko (2015). Information rigidity and the expectations formation process: A simple framework and new facts. *American Economic Review* 105(8), 2644–78.
- Coibion, O., Y. Gorodnichenko, and R. Kamdar (2018). The formation of expectations, inflation, and the phillips curve. *Journal of Economic Literature* 56(4), 1447–91.
- Erceg, C. J., D. W. Henderson, and A. T. Levin (2000, October). Optimal monetary policy with staggered wage and price contracts. *Journal of Monetary Economics* 46(2), 281–313.

- Galí, J. (2013). Notes for a new guide to keynes (i): wages, aggregate demand, and employment. *Journal of the European Economic Association* 11(5), 973–1003.
- Galí, J. and T. Monacelli (2016). Understanding the gains from wage flexibility: the exchange rate connection. *American Economic Review* 106(12), 3829–68.
- Hellwig, C. and V. Venkateswaran (2009). Setting the right prices for the wrong reasons. *Journal of Monetary Economics* 56, S57 – S77. Supplement issue: December 12-13, 2008 Research Conference on ‘Monetary Policy under Imperfect Information’ Sponsored by the Swiss National Bank (<http://www.snb.ch>) and Study Center Gerzensee (www.szgerzensee.ch).
- Huang, K. X. and Z. Liu (2005, November). Inflation targeting: What inflation rate to target? *Journal of Monetary Economics* 52(8), 1435–1462.
- Huo, Z. and M. Pedroni (2019). A Single-Judge Solution to Beauty Contests. pp. 75.
- Huo, Z. and N. Takayama (2015a). Higher order beliefs, confidence, and business cycles. *Report, Yale University*. [1, 2].
- Huo, Z. and N. Takayama (2015b). Rational expectations models with higher order beliefs. Technical report, Yale mimeo.
- Keynes, J. M. (1936). *The general theory of employment, interest, and money*. Springer.
- Long, J. B. D. and L. H. Summers (1986). Is increased price flexibility stabilizing? *The American Economic Review* 76(5), 1031–1044.
- Lorenzoni, G. (2009). A theory of demand shocks. *American Economic Review* 99(5), 2050–84.

- Lucas, R. E. (1972). Expectations and the neutrality of money. *Journal of Economic Theory* 4(2), 103–124.
- Mackowiak, B. and M. Wiederholt (2009). Optimal sticky prices under rational inattention. *American Economic Review* 99(3), 769–803.
- Mankiw, N. G. and R. Reis (2002). Sticky information versus sticky prices: a proposal to replace the new keynesian phillips curve. *The Quarterly Journal of Economics* 117(4), 1295–1328.
- Mankiw, N. G. and R. Reis (2003, September). What Measure of Inflation Should a Central Bank Target? *Journal of the European Economic Association* 1(5), 1058–1086.
- Melosi, L. (2016). Signalling effects of monetary policy. *The Review of Economic Studies* 84(2), 853–884.
- Morris, S. and H. S. Shin (2002). Social value of public information. *American Economic Review* 92(5), 1521–1534.
- Nakamura, E. and J. Steinsson (2008). Five facts about prices: A reevaluation of menu cost models. *The Quarterly Journal of Economics* 123(4), 1415–1464.
- Nimark, K. (2008). Dynamic pricing and imperfect common knowledge. *Journal of Monetary Economics* 55(2), 365–382.
- Rotemberg, J. J. and M. Woodford (1997). An optimization-based econometric framework for the evaluation of monetary policy. *NBER macroeconomics annual* 12, 297–346.
- Rotemberg, J. J. and M. Woodford (1999). Interest rate rules in an estimated sticky price model. In *Monetary policy rules*, pp. 57–126. University of Chicago Press.

- Sims, C. A. (2003). Implications of rational inattention. *Journal of monetary Economics* 50(3), 665–690.
- Veldkamp, L. L. (2011). *Information choice in macroeconomics and finance*. Princeton University Press.
- Woodford, M. (2001). Imperfect common knowledge and the effects of monetary policy. Technical report, National Bureau of Economic Research.
- Woodford, M. (2002). Inflation stabilization and welfare. *Contributions in Macroeconomics* 2(1).
- Woodford, M. (2011). *Interest and prices: Foundations of a theory of monetary policy*. princeton university press.
- Zhang, D. (2018). Which inflation index to stabilize? the role of market power. Technical report, Mimeo.

A Appendix

A.1 Figures

Figure 8: Welfare Losses Decomposition: Perfect Information

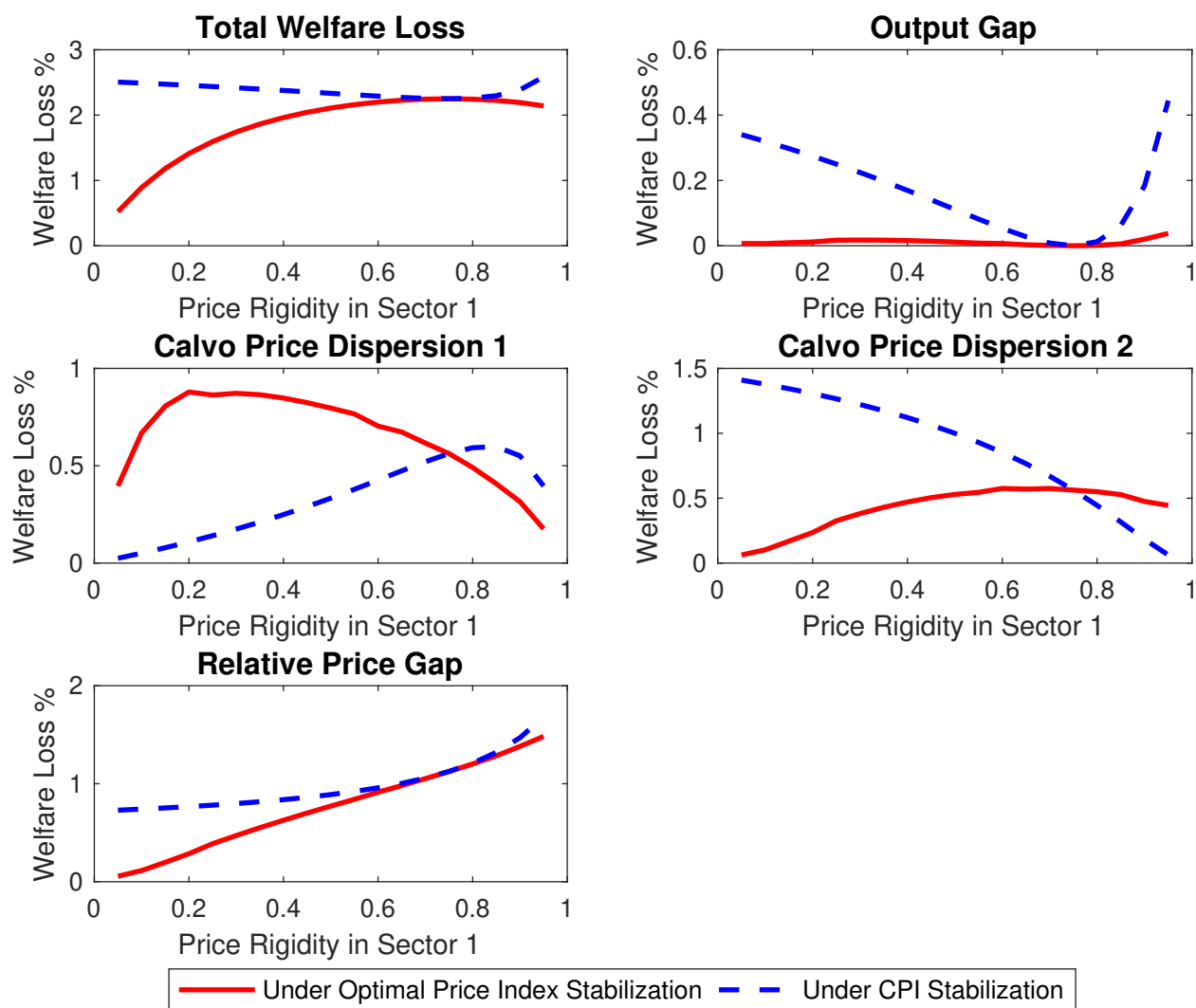
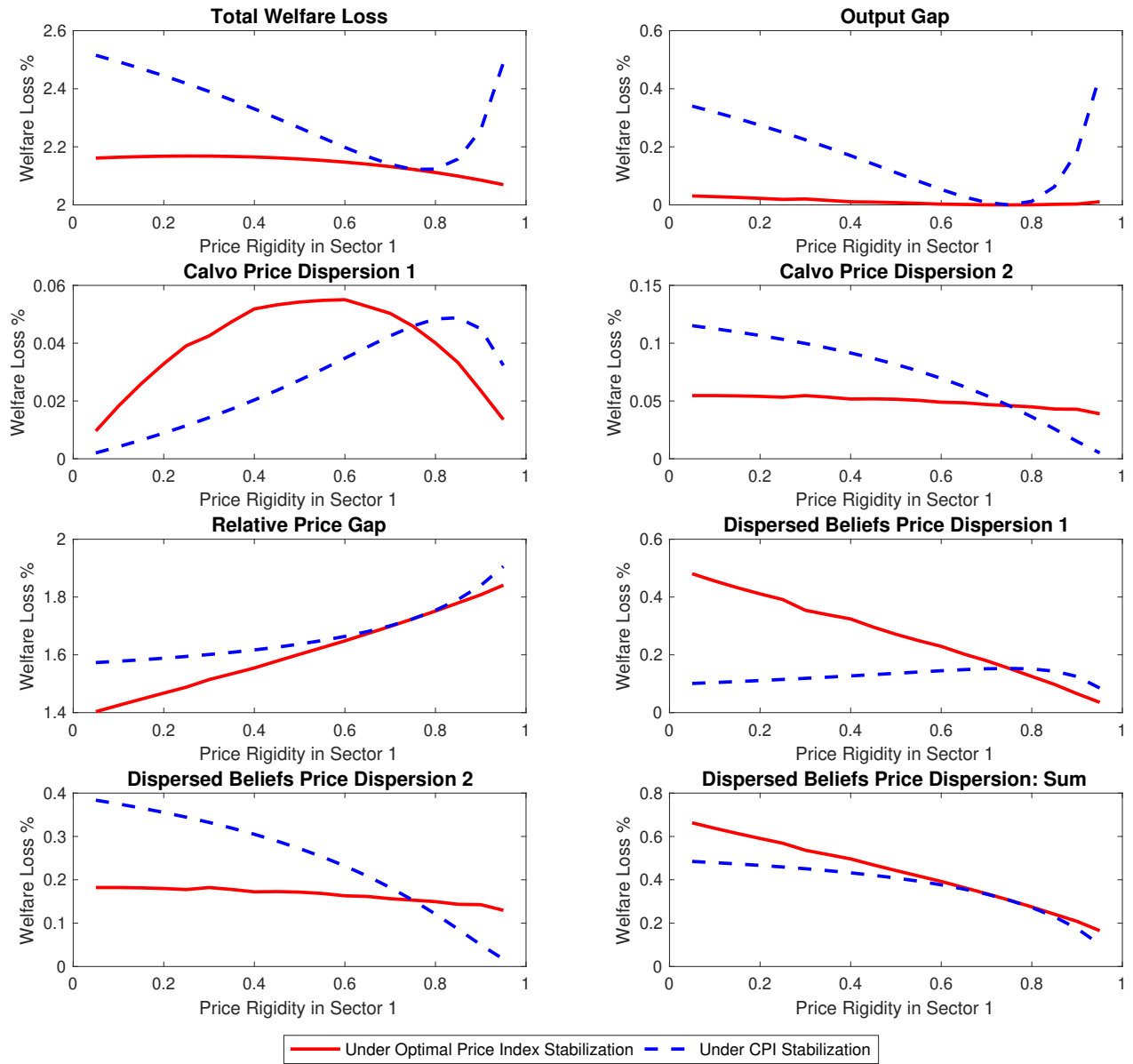


Figure 9: Welfare Losses Decomposition: Dispersed Beliefs



A.2 Welfare Loss function

The second order Taylor expansion of the representative household's utility U_t around a steady-state (C, L) in terms of log deviations can be written as:

$$U_t - U \approx U_c C \left(\hat{y}_t + \frac{1-\sigma}{2} \hat{y}_t^2 \right) + \sum_{k=1}^K U_{L_k} L_k \left(\hat{l}_{kt} + \frac{1+\varphi}{2} \hat{l}_{kt}^2 \right) di.$$

Note that

$$\hat{l}_{kt} = \hat{y}_{kt} - a_{kt} + d_{kt}$$

where $d_{kt} \equiv \log\left(\frac{P_{kt}(i)}{P_{kt}}\right) - \epsilon$.

Lemma A.1. $d_{kt} = \frac{\epsilon}{2} \text{var}_i\{p_{kt}(i)\}$

Proof: Gali (2008, chapter 4)

Therefore,

$$U_t - U \approx U_c C \left(\hat{y}_t + \frac{1-\sigma}{2} \hat{y}_t^2 \right) + \sum_{k=1}^K U_{L_k} L_k \left(\hat{y}_{kt} + \frac{\epsilon}{2} \text{var}_i\{p_{kt}(i)\} + \frac{1+\varphi}{2} (\hat{y}_{kt} - a_{kt})^2 \right) + t.i.p.$$

where *t.i.p* denotes the terms independent of policy. Under the assumption that cost of employment is subsidized optimally at sectorial level to eliminate distortions originate from monopolistic competition, the steady-state is efficient and $-\frac{U_{L_k}}{U_c} = MPN$.

Approximate the aggregate C_t around $c_k = c + \log(n_k)$:

$$\sum_{k=1}^K n_k \hat{y}_{kt} \approx \hat{y}_t - \frac{1-\eta^{-1}}{2} \sum_{k=1}^K n_k \hat{y}_{R,kt}^2$$

with $\sum_{k=1}^K n_k \hat{y}_{R,kt}^2 \equiv \sum_{k=1}^K n_k (\hat{y}_{kt} - \hat{y}_t)^2$. Using the fact that $MPN = (Y_k/L_k), Y = C$, it follows that:

$$\begin{aligned}
\frac{U_t - U}{U_c C} &\approx -\frac{1}{2} \left[\sum_{k=1}^K \left(\epsilon n_k \text{var}_i \{p_{kt}(i)\} \right) - (1 - \sigma) \hat{y}_t^2 - (1 - \eta^{-1}) \sum_{k=1}^K n_k \hat{y}_{R,kt}^2 \right. \\
&\quad \left. + (1 + \varphi) \sum_{k=1}^K n_k (\hat{y}_{kt} - a_{kt})^2 \right] + t.i.p \\
&= -\frac{1}{2} \left[\sum_{k=1}^K \left(\epsilon n_k \text{var}_i \{p_{kt}(i)\} \right) - (1 - \sigma) \hat{y}_t^2 - (1 - \eta^{-1}) \sum_{k=1}^K n_k \hat{y}_{R,kt}^2 \right. \\
&\quad \left. + (1 + \varphi) \sum_{k=1}^K n_k (\hat{y}_{kt}^2 - 2\hat{y}_{kt} a_{kt}) \right] + t.i.p \\
&= -\frac{1}{2} \left[\sum_{k=1}^K \left(\epsilon n_k \text{var}_i \{p_{kt}(i)\} \right) + (\sigma + \varphi) \hat{y}_t^2 + (\eta^{-1} + \varphi) \sum_{k=1}^K n_k \hat{y}_{R,kt}^2 \right. \\
&\quad \left. - 2(1 + \varphi) \sum_{k=1}^K n_k \hat{y}_{kt} a_{kt} \right] + t.i.p \\
&= -\frac{1}{2} \left[\sum_{k=1}^K \left(\epsilon n_k \text{var}_i \{p_{kt}(i)\} \right) + (\sigma + \varphi) \hat{y}_t^2 + (\eta^{-1} + \varphi) \sum_{k=1}^K n_k \hat{y}_{R,kt}^2 \right. \\
&\quad \left. - 2(1 + \varphi) \sum_{k=1}^K n_k (\hat{y}_t + \hat{y}_{kt} - \hat{y}_t) a_{kt} \right] \\
&= -\frac{1}{2} \left[\sum_{k=1}^K \left(\epsilon n_k \text{var}_i \{p_{kt}(i)\} \right) + (\sigma + \varphi) \hat{y}_t^2 + (\eta^{-1} + \varphi) \sum_{k=1}^K n_k \hat{y}_{R,kt}^2 \right. \\
&\quad \left. - 2(\sigma + \varphi) \hat{y}_t y^N - 2(\eta^{-1} + \varphi) \sum_{k=1}^K n_k (\hat{y}_{kt} - y_t) (\hat{y}_{kt}^N - y_t^N) \right] + t.i.p \\
&= -\frac{1}{2} \left[\sum_{k=1}^K \left(\epsilon n_k \text{var}_i \{p_{kt}(i)\} \right) + (\sigma + \varphi) \hat{y}_t^2 + (\eta^{-1} + \varphi) \sum_{k=1}^K n_k \hat{y}_{R,kt}^2 \right] + t.i.p
\end{aligned}$$

where $\tilde{y}_t \equiv y_t - y_t^N$. From line 2 to line 3, I have used the fact that $\sum_{k=1}^K n_k \hat{y}_{kt}^2 = \sum_{k=1}^K n_k \hat{y}_{R,kt}^2 + (\sum_{k=1}^K n_k \hat{y}_{kt})^2 \approx \sum_{k=1}^K n_k \hat{y}_{R,kt}^2 + \hat{y}_t^2$. From line 4 to line 5, we have used the fact that $\sum_{k=1}^K n_k a_{kt} = \frac{\sigma + \varphi}{1 + \varphi} y_t^N$ and $a_{kt} - \sum_{k=1}^K n_k a_{kt} = \frac{\eta^{-1} + \varphi}{1 + \varphi} (y_{kt}^N - y_t^N)$.

To summarize, the second order approximation of the representative consumer's pe-

riod welfare loss as a fraction of steady-state consumption is:

$$WL = \sum_{k=1}^K \left(\epsilon n_k \text{var}_i \{ p_{kt}(i) \} \right) + (\sigma + \varphi) \tilde{y}_t^2 + (\eta^{-1} + \varphi) \sum_{k=1}^K n_k \tilde{y}_{R,kt}^2 \quad (\text{A.1})$$

$$= \sum_{k=1}^K \left(\epsilon n_k \text{var}_i \{ p_{kt}(i) \} \right) + (\sigma + \varphi) \tilde{y}_t^2 + (\eta^{-1} + \varphi) \eta^2 \sum_{k=1}^K n_k \tilde{p}_{R,kt}^2 \quad (\text{A.2})$$

Next, we will show

$$\text{var}_i \{ p_{ki,t} \} = (1 - \theta_k) \int_i (p_{ki,t}^* - p_{k,t}^*)^2 di + \frac{\theta_k}{1 - \theta_k} \text{var}(\pi_{k,t}). \quad (\text{A.3})$$

Following Let $\Delta_{k,t} \equiv \text{var}_i \{ p_{ki,t} \}$

$$\Delta_{k,t} = \theta_k \Delta_{k,t-1} + (1 - \theta_k) \int_i (p_{ki,t}^* - p_{k,t}^* + p_{k,t}^* - p_{k,t-1})^2 di - \pi_{k,t}^2 \quad (\text{A.4})$$

$$= \theta_k \Delta_{k,t-1} + (1 - \theta_k) \int_i (p_{ki,t}^* - p_{k,t}^*)^2 di + (1 - \theta_k) \int_i (p_{k,t}^* - p_{k,t-1})^2 di - \pi_{k,t}^2 \quad (\text{A.5})$$

$$= \theta_k \Delta_{k,t-1} + (1 - \theta_k) \int_i (p_{ki,t}^* - p_{k,t}^*)^2 di + \frac{\theta_k}{1 - \theta_k} \pi_{k,t}^2. \quad (\text{A.6})$$

Therefore,

$$\sum_{t=0}^{\infty} \text{var}_i \{ p_{ki,t} \} = \frac{1}{1 - \beta \theta_k} \left[\sum_{t=0}^{\infty} (1 - \theta_k) \int_i (p_{ki,t}^* - p_{k,t}^*)^2 di + \frac{\theta_k}{1 - \theta_k} \pi_{k,t}^2 \right].$$

Welfare loss function in the static model The counterpart of equation A.2 in the static model is:

$$WL = \sum_{k=1}^K \left(\epsilon n_k \text{var}_i \{ p_{k,i} \} \right) + \sigma x^2 + \eta \sum_{k=1}^K n_k \tilde{p}_{R,k}^2 \quad (\text{A.7})$$

where x denotes output gap in the static model.

$$\begin{aligned}
var_i p_{k,i} &= \int_i (p_{k,i} - p_k)^2 di \\
&= (1 - \theta_k) \int_i (p_{ki}^* - p_k)^2 di + \theta_k p_k^2 \\
&= (1 - \theta_k) \left(\int_i (p_{ki}^* - p_k^*)^2 di + (p_k^* - p_k)^2 \right) + \theta_k p_k^2 \\
&= (1 - \theta_k) \phi_k^2 \sigma_e^2 + (1 - \theta_k) \theta_k p_k^{*2}
\end{aligned}$$

from the second to the third equality, we have used (??). Note that in the two-sector model $\sum_{k=1}^2 n_k \tilde{p}_{R,k}^2 = 2n_1 n_2 (\tilde{p}_1 - \tilde{p}_2)^2$, therefore, the period welfare loss function in the static model is:

$$\mathbb{E} \epsilon \sum_{k=1}^2 n_k \left((1 - \theta_k) \phi_k^2 \sigma_e^2 + (1 - \theta_k) \theta_k p_k^{*2} \right) + \sigma x^2 + 2\eta n_1 n_2 (\tilde{p}_1 - \tilde{p}_2)^2 \quad (\text{A.8})$$

A.3 Solution Method for the Static Model

In this section, we solve the static model with heterogeneous and imperfect information using the method of the undetermined coefficient. The method is shown in a general way, which allows u_1 and u_2 not to be perfectly linearly dependent on each other and each firm receives signals for both u_1 and u_2 .

A.3.1 Solution method

We follow the solution first discussed by [Morris and Shin \(2002\)](#), and [Veldkamp \(2011\)](#) provides a textbook treatment. Given the quadratic loss function and linear constraints,

we conjecture that the output gap will take the following form (our initial guess),

$$x = \beta_1 u_1 + \beta_2 u_2. \quad (\text{A.9})$$

Step1: solving the firm's belief updating problem. A firm i in sector 1 updates her belief based on the Bayes' law,

$$\mathbb{E}_{1i} \begin{pmatrix} x \\ u_1 \\ u_2 \end{pmatrix} = G \begin{pmatrix} s_{u1,1}(i) \\ s_{u2,1}(i) \end{pmatrix} = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \\ G_{31} & G_{32} \end{pmatrix} \begin{pmatrix} s_{u1,1}(i) \\ s_{u2,1}(i) \end{pmatrix}, \quad (\text{A.10})$$

with each entry of G defined as $\frac{\text{cov}(s,x)}{\text{var}(s)}$, where $\text{cov}(s,x)$ is the covariance of the unobserved fundamental and the observed signal, and $\text{var}(s)$ is the variance of the signal. In particular,

$$\begin{aligned} G_{11} &= \frac{\text{cov}(x, s_{u1,1}(i))}{\text{var}(s_{u1,1}(i))} = \frac{\beta_1 \sigma_{u1}^2}{\sigma_{u1}^2 + \sigma_{u1,1}^2} \\ G_{12} &= \frac{\text{cov}(x, s_{u2,1}(i))}{\text{var}(s_{u2,1}(i))} = \frac{\beta_2 \sigma_{u2}^2}{\sigma_{u2}^2 + \sigma_{u2,1}^2}. \end{aligned}$$

Similarly, a firm i in sector 2 updates her belief according to:

$$\mathbb{E}_{2i} \begin{pmatrix} x \\ u_1 \\ u_2 \end{pmatrix} = H \begin{pmatrix} s_{u1,2}(i) \\ s_{u2,2}(i) \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \\ H_{31} & H_{32} \end{pmatrix} \begin{pmatrix} s_{u1,2}(i) \\ s_{u2,2}(i) \end{pmatrix} \quad (\text{A.11})$$

where

$$\begin{aligned} H_{11} &= \frac{\text{cov}(x, s_{u1,2}(i))}{\text{var}(s_{u1,2}(i))} = \frac{\beta_1 \sigma_{u1}^2}{\sigma_{u1}^2 + \sigma_{u1,2}^2} \\ H_{12} &= \frac{\text{cov}(x, s_{u2,2}(i))}{\text{var}(s_{u2,2}(i))} = \frac{\beta_2 \sigma_{u2}^2}{\sigma_{u2}^2 + \sigma_{u2,2}^2}. \end{aligned}$$

Define $Z \equiv \begin{pmatrix} x \\ u_1 \\ u_2 \end{pmatrix}$, $S_1 \equiv \begin{pmatrix} s_{u1,1}(i) \\ s_{u2,1}(i) \end{pmatrix}$, $S_2 \equiv \begin{pmatrix} s_{u1,2}(i) \\ s_{u2,2}(i) \end{pmatrix}$, and denote the rows of G and H by $G_1, G_2, G_3, H_1, H_2, H_3$ respectively. Then equation (A.10) and (A.11) can be rewritten more compactly as:

$$\mathbb{E}_{1i} Z = G S_1$$

$$\mathbb{E}_{2i} Z = H S_2.$$

Define $g \equiv \begin{pmatrix} G_{21} & G_{22} \\ G_{31} & G_{32} \end{pmatrix}$, $h \equiv \begin{pmatrix} H_{21} & H_{22} \\ H_{31} & H_{32} \end{pmatrix}$, and $U \equiv \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ that will be used below to simplify notations.

Step 2: Recall that the individual optimal price-setting decision follows

$$p_{1,i}^* = \mathbb{E}_{1i}[n_1 p_1 + n_2 p_2 + \alpha x + u_1], \quad (\text{A.12})$$

$$p_{2,i}^* = \mathbb{E}_{2i}[n_1 p_1 + n_2 p_2 + \alpha x + u_2] \quad (\text{A.13})$$

Hence, we should obtain the representations for $\mathbb{E}_{ki} p_1$, $\mathbb{E}_{ki} p_2$, $\mathbb{E}_{ki} x$, $\mathbb{E}_{ki} u_1$ for $k = 1, 2$, which is what step 3 presents.

Step 3: Conjecture that

$$p_{1,i}^* = \phi_{11}s_{u1,1}(i) + \phi_{12}s_{u2,1}(i) \quad (\text{A.14})$$

$$= \Phi_1 S_1 \quad (\text{A.15})$$

$$p_{2,i}^* = \phi_{21}s_{u1,2}(i) + \phi_{22}s_{u2,2}(i) \quad (\text{A.16})$$

$$= \Phi_2 S_2 \quad (\text{A.17})$$

where Φ_1 stands for $\begin{pmatrix} \phi_{11} \\ \phi_{12} \end{pmatrix}'$, Φ_2 for $\begin{pmatrix} \phi_{21} \\ \phi_{22} \end{pmatrix}'$. Since the means of signals are the true states, our conjectures imply the following aggregate price-setting behavior,

$$p_1^* = \phi_{11}u_1 + \phi_{12}u_2 \quad (\text{A.18})$$

$$= \Phi_1 U \quad (\text{A.19})$$

$$p_2^* = \phi_{21}u_1 + \phi_{22}u_2 \quad (\text{A.20})$$

$$= \Phi_2 U \quad (\text{A.21})$$

Let $\lambda_k = (1 - \theta_k)$ to simplify notation. Thus, the individual firm's expectations of average sectoral prices are

$$E_{1i}(p_1) = \lambda_1 E_{1i}(p_1^*) = \lambda_1 [\phi_{11} E_{1i}(u_1) + \phi_{12} E_{1i}(u_2)] \quad (\text{A.22})$$

$$= \lambda_1 \Phi_1 E_{1i}(U) = \lambda_1 \Phi_1 g S_1 \quad (\text{A.23})$$

$$E_{1i}(p_2) = \lambda_2 E_{1i}(p_2^*) = \lambda_2 [\phi_{21} E_{1i}(u_1) + \phi_{22} E_{1i}(u_2)] \quad (\text{A.24})$$

$$= \lambda_2 \Phi_2 E_{1i}(U) = \lambda_2 \Phi_2 g S_1 \quad (\text{A.25})$$

$$E_{2i}(p_1) = \lambda_1 E_{2i}(p_1^*) = \lambda_1 [\phi_{11} E_{2i}(u_1) + \phi_{12} E_{2i}(u_2)] \quad (\text{A.26})$$

$$= \lambda_1 \Phi_1 E_{2i}(U) = \lambda_1 \Phi_1 h S_2 \quad (\text{A.27})$$

$$E_{2i}(p_2) = \lambda_2 E_{2i}(p_2^*) = \lambda_1 [\phi_{23} E_{1i}(u_1) + \phi_{24} E_{1i}(u_2)] \quad (\text{A.28})$$

$$= \lambda_2 \Phi_2 E_{2i}(U) = \lambda_2 \Phi_2 h S_2 \quad (\text{A.29})$$

Similarly, we have

$$E_{1i}(x) = G_1 S_1 \quad (\text{A.30})$$

$$E_{1i}(u_1) = G_2 S_1 \quad (\text{A.31})$$

$$E_{2i}(x) = H_1 S_2 \quad (\text{A.32})$$

$$E_{2i}(u_2) = H_3 S_2. \quad (\text{A.33})$$

Step 4: Plugging equation (A.22) to (A.33) to the right hand side of equation (A.12) and equation (A.13) gives rise to

$$\Phi_1 S_1 = n_1 \lambda_1 \Phi_1 g S_1 + n_2 \lambda_2 \Phi_2 g S_1 + \alpha G_1 S_1 + G_2 S_1 \quad (\text{A.34})$$

$$\Phi_2 S_2 = n_1 \lambda_1 \Phi_1 h S_2 + n_2 \lambda_2 \Phi_2 h S_2 + \alpha H_1 S_2 + H_3 S_2. \quad (\text{A.35})$$

The above two equations need to hold for all S_1 and S_2 respectively. Collecting terms involving same factors leads to

$$\Phi_1 = n_1 \lambda_1 \Phi_1 g + n_2 \lambda_2 \Phi_2 g + \alpha G_1 + G_2 \quad (\text{A.36})$$

$$\Phi_2 = n_1 \lambda_1 \Phi_1 h + n_2 \lambda_2 \Phi_2 h + \alpha H_1 + H_3 \quad (\text{A.37})$$

And we have

$$\omega \lambda_1 \Phi_1 = -(1 - \omega) \lambda_2 \Phi_2$$

led by $0 = \omega p_1 + (1 - \omega) p_2$.

Note that G_1 and H_1 are actually a function of policy coefficients β_1 and β_2 while G_2 and H_3 only consist of parameters. Implicitly, we have 6 equations for 6 unknowns ϕ_{11} ϕ_{12} ϕ_{21} ϕ_{22} β_1 β_2 . We further simplify the above system of equations to the follows equations

$$\begin{aligned}\Phi_1 = & [n_2\lambda_2(\alpha H_1 + \alpha H_3)(I - n_2\lambda_2h)^{-1}g + \alpha G_1 + G_2] \\ & \cdot [I - \theta_2\lambda_2n_1\lambda_1h(I - \theta_2\lambda_2h)^{-1}g - n_1\lambda_1g]^{-1}\end{aligned}\quad (\text{A.38})$$

$$\begin{aligned}\Phi_2 = & [n_1\lambda_1(\alpha G_1 + \alpha G_2)(I - n_1\lambda_1g)^{-1}h + \alpha H_1 + H_3] \\ & \cdot [I - n_1\lambda_1n_2\lambda_2g(I - n_1\lambda_1g)^{-1}h - n_2\lambda_2h]^{-1}\end{aligned}\quad (\text{A.39})$$

$$\omega\lambda_1\Phi_1 = -(1 - \omega)\lambda_2\Phi_2 \quad (\text{A.40})$$

where I is a 2 by 2 identity matrix. Thus we have verified that our initial guesses are correct and show how to get the solution for the pricing function. Moreover, it is shown by [Morris and Shin \(2002\)](#) that the solution in this type of model is unique. For readers who wonder how high-order beliefs are simplified in this framework, this is due to two reasons: first, the optimal price-setting function of individual firms are common knowledge and second expectations of the other's beliefs rely solely on the expectations of the fundamentals.

step 5: the optimal ω The optimal price target is achieved by choosing the optimal weight ω to minimize the welfare loss function.

A.4 Solution Method for Dynamic Model

We follow the method developed by [Nimark \(2008\)](#) and [Melosi \(2016\)](#) to solve the dynamic model.

A.4.1 The Undetermined Coefficients Method

As is in [Nimark \(2008\)](#) and [Melosi \(2016\)](#), the law of motion of the endogenous variables

$\mathbf{s}_t \equiv [\tilde{\mathbf{y}}_t, \{\hat{\mathbf{y}}_{R,kt}\}_{k=1}^K, \{\pi_{kt}\}_{k=1}^K]'$ reads as follows:

$$\mathbf{s}_t = \mathbf{v}_0 X_{t|t}^{(0:J)}$$

where $X_{t|t}^{(0:J)} \equiv [\{a_{k,t|t}^{(s)}\}_{k=1}^K, \{\hat{\mathbf{y}}_{R,kt-1}\}_{k=1}^{K-1} : 0 \leq s \leq J]$ are the truncated state space as a $K(J+2) - 1$ by 1 vector. Note that due to co-linearity $\hat{\mathbf{y}}_{R,Kt} = -\sum_{k=1}^{K-1} n_k \hat{\mathbf{y}}_{R,kt}$, we have deleted $\hat{\mathbf{y}}_{R,Kt-1}$ from the state variables space and deleted $\hat{\mathbf{y}}_{R,Kt}$ from choice variables. We move directly to the truncated state space approximation rather than showing the original infinite high-order expectation case to facilitate the description of the solution algorithm. Specifically,

$$\begin{aligned} \tilde{\mathbf{y}}_t &= \mathbf{a} X_{t|t}^{(0:J)} \\ \hat{\mathbf{y}}_{R,kt} &= \mathbf{b}_k X_{t|t}^{(0:J)} \\ \pi_{kt} &= \mathbf{c}_k X_{t|t}^{(0:J)} \end{aligned}$$

and

$$\mathbf{v}_0 = [\mathbf{a}; \mathbf{b}_1; \dots; \mathbf{b}_{K-1}; \mathbf{c}_1; \dots; \mathbf{c}_K]$$

where ; represents another row and these policy function coefficients are not determined yet.

As in [Nimark \(2008\)](#) and [Melosi \(2016\)](#), the law of motion of state variables follows

$$X_{t|t}^{(0:J)} = M X_{t-1|t-1}^{(0:J)} + N \mathbf{f} \mathbf{f}_t$$

where $\mathbf{ffl}_t = [\epsilon_{1t} \dots \epsilon_{Kt}]'$, M is $K(J+2) - 1$ by $K(J+2) - 1$ matrix and N is $K(J+2) - 1$ by K matrix. M and N will be determined by the Kalman filter. Define matrix $T^{(s)}$ as follows:

$$T^{(s)} = \begin{bmatrix} \mathbf{0}_{K(J-s+1) \times Ks} & I_{K(J-s+1)} & \mathbf{0}_{K(J-s+1) \times (K-1)} \\ \mathbf{0}_{Ks \times Ks} & \mathbf{0}_{Ks \times K(J-s+1)} & \mathbf{0}_{Ks \times (K-1)} \\ \mathbf{0}_{(K-1) \times Ks} & \mathbf{0}_{(K-1) \times K(J-s+1)} & I_{K-1} \end{bmatrix}$$

where $I_{K(J-s+1)}$ represents $K(J-s+1)$ by $K(J-s+1)$ identity matrix. Then we have the following important results

$$\begin{aligned} \mathbf{s}_{t|t}^{(s)} &= \mathbf{v}_0 T^{(s)} X_{t|t}^{(0:J)} \\ \mathbf{s}_{t+h|t}^{(s)} &= \mathbf{v}_0 M^h T^{(s)} X_{t|t}^{(0:J)} \end{aligned}$$

for any $0 \leq s \leq J$.

Define $\epsilon_k^{(s)} \equiv [\mathbf{0}_{1 \times Ks}, \{-n_m\}_{m=1}^{k-1}, (1-n_k), \{-n_m\}_{m=k+1}^K, \mathbf{0}_{1 \times K(J+1-s)-1}]$. Substituting the policy function to the (3.8) after the truncation leads to

$$\begin{aligned} \mathbf{c}_k X_{t|t}^{(0:J)} &= (1-\theta)(1-\beta\theta)\Theta \sum_{j=1}^{\infty} (1-\theta)^{j-1} [(\sigma + \varphi) \mathbf{a} T^{(j)} X_{t|t}^{(0:J)} \\ &\quad + (\eta^{-1} + \varphi)(\mathbf{b}_k T^{(j)} X_{t|t}^{(0:J)} - \Psi \epsilon_k^{(j)} X_{t|t}^{(0:J)})] + \beta\theta \sum_{j=1}^{\infty} (1-\theta)^{j-1} \mathbf{c}_k M T^{(j)} X_{t|t}^{(0:J)}. \end{aligned}$$

which implies a restriction on the policy function coefficients

$$\begin{aligned}
\mathbf{c}_k = & (1 - \theta)(1 - \beta\theta)\Theta \sum_{j=1}^J (1 - \theta)^{j-1} [(\sigma + \varphi)\mathbf{a}T^{(j)} + (\eta^{-1} + \varphi)(\mathbf{b}_k T^{(j)} - \Psi\epsilon_k^{(j)})] \\
& + \beta\theta \sum_{j=1}^J (1 - \theta)^{j-1} \mathbf{c}_k MT^{(j)}
\end{aligned} \tag{A.41}$$

Define $A_k \equiv [1 \ (1 - \theta_k) \ (1 - \theta_k)^2 \dots (1 - \theta_k)^{J-1}]$ and let

$$V \equiv \begin{bmatrix} (\sigma + \varphi)\mathbf{a}T^{(1)} + (\eta^{-1} + \varphi)(\mathbf{b}_k T^{(1)} - \Psi\epsilon_k^{(1)}) \\ \vdots \\ (\sigma + \varphi)\mathbf{a}T^{(J)} + (\eta^{-1} + \varphi)(\mathbf{b}_k T^{(J)} - \Psi\epsilon_k^{(J)}) \end{bmatrix}$$

and

$$H \equiv \begin{bmatrix} \mathbf{c}_k MT^{(1)} \\ \vdots \\ \mathbf{c}_k MT^{(J)} \end{bmatrix}.$$

Then equation (A.41) can be rewritten more compactly as

$$\mathbf{c}_k = (1 - \theta)(1 - \beta\theta)\Theta A_k V + \beta\theta A H$$

Similarly, we have

$$\mathbf{a}X_{t|t}^{(0:J)} = \mathbf{a}MT^{(1)}X_{t|t}^{(0:J)} - \frac{1}{\sigma}(\phi_\pi(\sum_{k=1}^K \omega_k \mathbf{c}_k X_{t|t}^{(0:J)})) - \sum_{k=1}^K n_k \mathbf{c}_k MT^{(1)}X_{t|t}^{(0:J)} + \sigma\psi^a \sum_{k=1}^K n_k (1 - \rho_k) a_{kt} \tag{A.42}$$

by substituting the policy function to IS curve.

Define matrix G by $G \equiv [\{n_k(1 - \rho_k)\}_{k=1}^K \mathbf{0}_{1 \times K(J+1)-1}]$. Then we have another restriction to the policy function coefficients,

$$\mathbf{a} = \mathbf{a}MT^{(1)} - \frac{1}{\sigma}(\phi_\pi(\sum_{k=1}^K \omega_k \mathbf{c}_k) - \sum_{k=1}^K n_k \mathbf{c}_k MT^{(1)} + \sigma \psi^a G). \quad (\text{A.43})$$

Define $\chi_k \equiv [\mathbf{0}_{1 \times K(J+1)}, \mathbf{0}_{1 \times k-1}, 1, \mathbf{0}_{1 \times (K-k)-1}]$. Similarly, for the law of motion of relative output, we have

$$\mathbf{b}_k X_{t|t}^{(0:J)} = -\eta(\mathbf{c}_k X_{t|t}^{(0:J)} - \sum_{k=1}^K n_k \mathbf{c}_k X_{t|t}^{(0:J)}) + \chi_k X_{t|t}^{(0:J)} \quad (\text{A.44})$$

which leads to the third restriction to the policy function coefficients

$$\mathbf{b}_k = -\eta(\mathbf{c}_k - \sum_{k=1}^K n_k \mathbf{c}_k) + \chi_k \quad (\text{A.45})$$

for $k \neq K$.

In summary, (A.41) (A.43) and (A.45) are the three equations that characterize the restrictions on policy functions given the law of motion for the state which in turn are governed by the Kalman filter and the policy functions described in a moment. Given the law of motion for the state, the policy functions can be solved by iteration easily.

A.4.2 Signal Extraction

Now we describe how the firms update their beliefs about the state and how the law of motion for the state are pinned down. Basically, given the law of motion for the state, we can apply the Kalman filter to derive a new expression for the law of motion for the state. Equating the new one with the previous one gives an updating rule to update the law of motion for the state. Moreover, the law of motion for state intervenes with the policy

functions, which calls for a fixed point problem to solve for the law of motion for the state and the policy functions simultaneously. Let's proceed with some notation to facilitate exposition.

Define $Z_{lt}(i) \equiv [\{\hat{y}_{R,k\tau-1}\}_{k=1}^{K-1}, \{s_{k,l\tau}(i)\}_{k=1}^K]'$ as signals received by firm i in sector l in period t , $\mathbf{c} \equiv [\mathbf{c}_1; \mathbf{c}_2; \dots; \mathbf{c}_K]$ as K by $K(J+2) - 1$ matrix, $\mathbf{b} \equiv [\mathbf{b}_1; \mathbf{b}_2; \dots; \mathbf{b}_{K-1}]$ as $K-1$ by $K(J+2) - 1$ matrix, and $\mathbf{e}_{lt}(i) \equiv [\{e_{k,l,t}(i)\}_{k=1}^K]'$ as K by 1 vector.

The observation equation with the lagged relative output as signals is

$$Z_{lt}(i) = L \begin{bmatrix} \mathbf{x}_t \\ \mathbf{x}_{t-1} \end{bmatrix} + Q\mathbf{e}_{lt}(i) \quad (\text{A.46})$$

where

$$L \equiv \begin{bmatrix} \mathbf{0}_{K-1 \times K(J+1)} & I_{K-1} & \mathbf{0}_{K-1 \times K(J+2)-1} \\ \mathbf{0}_{K \times K(J+1)} & \mathbf{0}_{K \times K-1} & \mathbf{c} \\ I_K & \mathbf{0}_{K \times K-1} & \mathbf{0}_{K \times K(2J+2)-1} \end{bmatrix}$$

$$Q \equiv \begin{bmatrix} \mathbf{0}_{K-1 \times K} \\ \mathbf{0}_{K \times K} \\ I_K \end{bmatrix}$$

$$\mathbf{x}_t \equiv \begin{bmatrix} X_{t|t}^{(0:J)} \end{bmatrix}$$

Recall that the law of motion for the state is

$$\mathbf{x}_t = M\mathbf{x}_{t-1} + N\mathbf{f}\mathbf{f}_t.$$

However, to implement the Kalman filter with the lagged state, we need to expand the state space as (A.46) hints. The law of motion for the expanded state follows

$$\mathcal{X}_t = W\mathcal{X}_{t-1} + \mathcal{N}\mathbf{f}\mathbf{f}_t \quad (\text{A.47})$$

where

$$\begin{aligned} W &= \begin{bmatrix} M_{K(J+2)-1 \times K(J+2)-1} & \mathbf{0}_{K(J+2)-1 \times K(J+2)-1} \\ I_{K(J+2)-1} & \mathbf{0}_{K(J+2)-1 \times K(J+2)-1} \end{bmatrix} \\ \mathcal{N} &= \begin{bmatrix} N_{K(J+2)-1 \times K} \\ \mathbf{0}_{K(J+2)-1 \times K} \end{bmatrix} \\ \mathcal{X}_t &= \begin{bmatrix} \mathbf{X}_t \\ \mathbf{X}_{t-1} \end{bmatrix}. \end{aligned}$$

Σ_{eel} and Σ_{ee} represent the variance-covariance matrix for the noise and for the sectoral TFP shock respectively. In summary, (A.46) and (A.47) consist of a linear space problem where the Kalman filter applies directly.

We use $\bar{K}$ to denote the stationary Kalman gain to differentiate from the sector size K . The steady-state Kalman filter gives

$$\mathcal{X}_{t|t,l}^{(1)}(i) = (W - \bar{K}LW)\mathcal{X}_{t-1|t-1,l}^{(1)}(i) + \bar{K}Z(i) \quad (\text{A.48})$$

$$= (W - \bar{K}LW)\mathcal{X}_{t-1|t-1,l}^{(1)}(i) + \bar{K}[LW\mathcal{X}_{t-1} + L\mathcal{N}\mathbf{f}\mathbf{f}_t + Q\mathbf{e}_{lt}(i)] \quad (\text{A.49})$$

where

$$\bar{K} = PL'(LPL' + \Sigma_{eel})^{-1}$$

with

$$P = W(P - PL'(LPL' + \Sigma_{el})^{-1}LP)W' + \Sigma_{\epsilon\epsilon}.$$

Denote the average of signals by Z_t such that $Z_t \equiv [\{\hat{y}_{R,k\tau-1}\}_{k=1}^{K-1}, \{a_k\}_{k=1}^K]' = \int_{i \in I} Z_{it}(i) di$ since the noise within each sector cancels out.

Taking average across (A.48) gives

$$\mathcal{X}_{t|t}^{(1)} = W\mathcal{X}_{t-1|t-1}^{(1)} + \bar{K} [Z_t - LW\mathcal{X}_{t-1|t-1}^{(1)}] \quad (\text{A.50})$$

$$= (W - \bar{K}LW)\mathcal{X}_{t-1|t-1}^{(1)} + \bar{K}[LW\mathcal{X}_{t-1}^{(1)} + L\mathcal{N}\mathbf{ffl}_t]. \quad (\text{A.51})$$

Now we want to insert the law of motion of the TFP shock into (A.51) to have a new expression for the law of motion for the state since (A.51) has excluded the zero-order expectation.

We can rewrite equation (A.51) as

$$\begin{aligned} \begin{bmatrix} \mathbf{x}_{t|t}^{(1)} \\ \mathbf{x}_{t-1|t}^{(1)} \end{bmatrix} &= \begin{bmatrix} (W - \bar{K}LW)_{11} & (W - \bar{K}LW)_{12} \\ (W - \bar{K}LW)_{21} & (W - \bar{K}LW)_{22} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{t-1|t-1}^{(1)} \\ \mathbf{x}_{t-2|t-1}^{(1)} \end{bmatrix} + \begin{bmatrix} \bar{K}LW_{11} & \bar{K}LW_{12} \\ \bar{K}LW_{21} & \bar{K}LW_{22} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{t-1} \\ \mathbf{x}_{t-2} \end{bmatrix} \\ &+ \begin{bmatrix} \bar{K}L\mathcal{N}_1 \\ \bar{K}L\mathcal{N}_2 \end{bmatrix} \mathbf{ffl}_t \end{aligned} \quad (\text{A.52})$$

where $(W - \bar{K}LW)_{12} = (W - \bar{K}LW)_{22} = 0$, $\bar{K}LW_{12} = \bar{K}LW_{22} = 0$ because of the construction of W .

Hence

$$\mathbf{X}_{t|t}^{(1)} = (W - \bar{K}LW)_{11} \mathbf{X}_{t-1|t-1}^{(1)} + \bar{K}LW_{11} \mathbf{X}_{t-1} + \bar{K}L\mathcal{N}_1 \mathbf{ffl}_t.$$

The $(J + 1)^{th}$ order of the average expectation which should be truncated by construction. Thus

$$\begin{aligned} \mathbf{X}_{t|t}^{(1)} \mid \sim_{(KJ+1:(K+1)J)} &= (W - \bar{K}LW)_{11} \mid \sim_{(KJ+1:(K+1)J)} \mathbf{X}_{t-1|t-1}^{(1)} \mid \sim_{(KJ+1:(K+1)J)} \\ &+ \bar{K}LW_{11} \mid \sim_{(KJ+1:(K+1)J)} \mathbf{X}_{t-1} + \bar{K}L\mathcal{N}_1 \mid \sim_{(KJ+1:(K+1)J)} \mathbf{ffl}_t. \end{aligned}$$

where $\mid \sim_{(KJ+1:(K+1)J)}$ with a bit of abuse of notation means deleting rows from row $KJ + 1$ to row $(K + 1)J$ for $\mathbf{X}_{t|t}^{(1)}$, $\mathbf{X}_{t-1|t-1}^{(1)}$ and $\bar{K}L\mathcal{N}_1$ while deleting both rows and columns for $(W - \bar{K}LW)_{11}$.

In order to simplify notation, we define $(W - \bar{K}LW)_{11}^* \equiv (W - \bar{K}LW)_{11} \mid \sim_{(KJ+1:(K+1)J)}$, $\bar{K}LW_{11}^* \equiv \bar{K}LW_{11} \mid \sim_{(KJ+1:(K+1)J)}$ and $\bar{K}L\mathcal{N}_1^* \equiv \bar{K}L\mathcal{N}_1 \mid \sim_{(KJ+1:(K+1)J)}$.

And recall that the sectoral TFP shocks evolve as

$$\begin{bmatrix} a_{1t} \\ \vdots \\ a_{Kt} \end{bmatrix} = \rho \begin{bmatrix} a_{1t-1} \\ \vdots \\ a_{Kt-1} \end{bmatrix} + I_K \mathbf{ffl}_t$$

where $\mathbf{a} \equiv \begin{bmatrix} \rho_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \rho_K \end{bmatrix}$ and I_K is a K by K matrix identity matrix.

Therefore, we can fully characterize the matrices M and N .

$$M = \begin{bmatrix} \mathbf{a} & \mathbf{0}_{K \times K(J+1)-1} \\ \mathbf{0}_{K(J+1)-1 \times K} & (W - \bar{K}LW)_{11}^* \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{K \times K(J+2)-1} \\ \bar{K}LW_{11}^* \end{bmatrix}$$

and

$$N = \begin{bmatrix} I_K \\ \mathbf{0}_{K(J+1)-1 \times K} \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{K \times K} \\ \bar{K}L\mathcal{N}_1^* \end{bmatrix}.$$

Note that the law of motion for the state depends on the policy functions and in turn, the policy functions depends on the law of motion for the state. In the end, we need to solve for a fixed point problem where the policy function and law of motion are jointly determined.