

Bubbly Firm Dynamics and Aggregate Fluctuations^{*}

Haozhou Tang^a, Donghai Zhang^{b,*}

^a*Institute for Advanced Economic Research, Dongbei University of Finance and Economics, Dalian, 116025, China*

^b*Institute for Macroeconomics and Econometrics, University of Bonn, Bonn, 53113, Germany*

Abstract

The transmission channel of asset bubbles is studied in a heterogeneous firm model with endogenous entry and exit. We highlight the effects of asset bubbles along the extensive margin: the aggregate bubble can boost real economic activities by affecting firms' entry and exit decisions. Moreover, the model predicts the selection effect of bubbles: bubbly firms—firms with asset bubbles—are less productive than bubble-less firms. Finally, we provide empirical evidence that supports bubbles' effects along the extensive margin.

Keywords: Rational Bubbles, Extensive Margin, Firm Dynamics, Heterogeneous Firms

JEL Classification: E32, E44, E47

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^{*}Corresponding author.

Email addresses: haozhoutang@dufe.edu.cn (Haozhou Tang), donghai.d.zhang@gmail.com (Donghai Zhang)

1. Introduction

Researchers have focused on the effects of asset bubbles on the aggregate economy because of the recent boom and bust of asset prices and their association with business cycles, e.g., [Farhi and Tirole \(2012\)](#), [Martin and Ventura \(2012\)](#), [Miao and Wang \(2018\)](#), and [Ikeda and Phan \(2019\)](#). These previous studies emphasize asset bubble's effects along the *intensive margin*: bubbles stimulate firms' production (e.g., by relaxing financial constraint) without affecting the number of firms. However, asset bubbles are linked to the values of firms, and therefore, entry and exit—the two most important decisions for firms—are linked to the acquisition/loss of bubbles.

The current paper focuses on the effects of asset bubbles along the *extensive margin*: asset bubbles affect the aggregate economy by influencing firms' entry and exit decisions. Is the extensive margin a relevant transmission channel of asset bubbles? What are the implications of asset bubbles for the distribution of firms? This study addresses these hitherto unaddressed questions through a theoretical model, and provides empirical evidence supporting the effects of asset bubbles along the extensive margin.

Theoretically, this study builds a heterogeneous firm model with endogenous entry and exit, allowing for rational bubbles. The framework helps in analyzing the effects of rational bubbles on real economic activities through firms' entry and exit decisions. The model nests the standard bubble-less firm dynamics model as a special case. Firms make entry (exit) decisions by comparing their value of operating in the market, which is composed of a fundamental and a bubble component, with the entry cost (outside option of exiting). Bubbles can influence decisions through their direct effects on firms' values. Specifically, for bubbly incumbents (i.e., existing firms whose value carries a bubble component), exiting the market leads to the loss of bubbles. In other words, bubbles act as an *exiting tax*, making it unattractive for a firm to exit. For potential entrants, bubbles act as an *entry subsidy*: ceteris paribus, the value of entering the market is higher because of an asset bubble. Hence, bubbles stimulate new firms to enter the market.

The calibrated model confirms the relevance of bubbles' effects along the extensive margin. Particularly, the total number of firms and aggregate output increase after a positive bubble shock (**Result 1**). Moreover, due to the extensive margin, the effects of an asset bubble on real economic activities are *slow-moving* and *hump shaped*, whereas bubble shocks affect asset prices *on impact* (**Result 2**).

Furthermore, bubbles have micro-level implications. Firms are heterogeneous in productivity, capital, and bubble component. We compare the characteristics of bubbly firms to their bubble-less counterparts along the balanced-growth path (BGP) in the model. On average, a bubbly firm is characterized by lower productivity and is shown to accumu-

late less capital than the corresponding variables of a bubble-less firm over the entire life cycle. The underlying intuition is that bubbles act as an exit tax and thus prevent low-productivity firms from exiting. Moreover, by acting as an entry subsidy, bubbles induce low-productivity firms to enter the market; therefore, bubbly firms are less productive on average, namely, the *selection effect* of bubbles. Consequently, bubbly firms accumulate less capital.

Finally, the paper provides empirical evidence that is consistent with our model by estimating a structural vector autoregressive (SVAR) model. Controlling for fundamental shocks, such as productivity, monetary policy, and risk premium shocks, we construct "bubble shocks" using the medium-run restriction, as pioneered by Uhlig (2003). The medium-run restriction identifies "bubble shocks" as the shocks that, among unobserved fundamental shocks that are not controlled for in the VAR, maximize the forecast error variance decomposition of our proxy of the non-fundamental component at the frequency of business cycles.¹

Using this empirical strategy, we uncover that an expansionary "bubble shock" increases the total number of firms by increasing the number of new firms and reducing firms' exit rates. Moreover, while the "bubble shock" has peak effects on asset prices and its non-fundamental component on impact (within a year), its effects on real economic activities are slow-moving and hump-shaped. These empirical findings are consistent with the effects of bubbles along the extensive margin that the model predicts, namely **Result 1** and **Result 2**. In addition, this paper provides evidence that the identified "bubble shock" is not confounded with fundamental shocks such as risk premium shocks, shocks to the short-term interest rate, and credit supply shocks. Last but not least, we demonstrate that bubble shocks can be a micro-foundation for empirically-identified news shocks.²

Literature Review. The study contributes to a growing body of literature on asset bubbles. Caballero and Krishnamurthy (2006), Farhi and Tirole (2012), Martin and Ventura (2012), Ventura (2012), Hirano and Yanagawa (2017), Miao and Wang (2018), Bengui and Phan (2018), Ikeda and Phan (2019), and Biswas et al. (2020) argue that bubbles relax financial constraints. Olivier (2000), and Queirós (2019) claim that imperfect competition in the goods market also allows bubbles to boost real economic activities. The existing works rely on market frictions for bubbles to boost real economic activities and focus on the

¹This paper uses the price-fundamental differential component and the non-fundamental component interchangeably to denote the deviations of an asset's price from its fundamental value. The fundamental component is defined as the present value of future real dividends.

²See, e.g., Barsky and Sims (2011) and Kurmann and Sims (2021) for the effects of news shocks identified by imposing the medium-run/max-share restriction on TFP.

effects of bubbles along the intensive margin. We emphasize a hitherto unexplored channel that gives rise to the co-movement of asset prices and real economic activities even in competitive markets. Moreover, we find the selection effect of bubbles through which bubbles affect the distribution of firm-level variables, such as productivity and capital.

This study is related to previous studies that emphasize the macroeconomic implications of firm-level heterogeneity. [Khan and Thomas \(2008\)](#), [Bachmann et al. \(2013\)](#), and [Winberry \(2020\)](#) study the implications of capital adjustment costs at the micro level for the behavior of aggregate investment. [Khan and Thomas \(2013\)](#), [Arellano et al. \(2019\)](#), and [Ottonello and Winberry \(2018\)](#) investigate the implications of financial frictions in models with heterogeneous firms. [Jaimovich and Floetotto \(2008\)](#), [Bilbiie et al. \(2012\)](#), [Lee and Mukoyama \(2015\)](#), [Clementi and Palazzo \(2016\)](#) and [Sedláček and Sterk \(2017\)](#) find that firm entry and/or exit justify important features of business cycles. In addition, [Jermann and Quadrini \(2007\)](#) relate asset price booms to aggregate productivity gains in a model with decreasing returns to scale and financial frictions. Our framework contributes to this literature by generalizing a firm dynamics model, allowing for rational bubbles. In our model, the bubble size is an additional dimension of firm heterogeneity. Through the selection effect, we show that bubbles can affect the distribution of firm-level productivity and capital.

The remainder of the paper is structured as follows. Section 2 provides a simple example of the existence of rational bubbles. Section 3 introduces the full-fledged model, and Section 4 presents the predictions of the model. Section 5 provides empirical evidence. Finally, Section 6 concludes.

2. An Example of Rational Bubbles

This section provides a simple example to illustrate bubbles' effects along the extensive margin. We show that, bubbles stimulate the aggregate output and productivity by increasing the number of firms even in the absence of market frictions.

2.1. Rational Bubbles

Setup. Consider a friction-less, small open economy with risk-neutral agents and a continuum of firms. Firm j produces using the technology $y_{jt} = \varphi_j k_{jt}^\alpha$. φ_j denotes the idiosyncratic productivity. Capital input k_{jt} is predetermined and fully depreciates during the production. Furthermore, the production process incurs a fixed cost c^f . Firms produce an identical product whose price is normalized to one.

We allow firms to enter and exit the goods market. First, there is a unit continuum of potential entrants every period with the prospect of becoming firms in the subsequent

period. Potential entrant j draws its productivity φ_j from differentiable distribution function $\Phi(\varphi)$. Second, in every period before production, firms receive an i.i.d. exit shock that destroys capital stock and forces firms to quit the market permanently, with the probability of $1 - p$. The value of survivors/potential entrants is given by

$$V_{jt} = \max_{k_{jt+1}} \left\{ -k_{jt+1} + \frac{p}{R} \left(y_{jt+1} - c^f + V_{jt+1} \right) \right\}, \quad (1)$$

where $R > p$ denotes the gross risk-free interest rate. Solving Equation (1) forward gives

$$V_{jt} = \sum_{\tau=t}^{\infty} \left(\frac{p}{R} \right)^{\tau-t} \left[-k_{j\tau+1} + \frac{p}{R} \left(\varphi_j k_{j\tau+1}^\alpha - c^f \right) \right] + \lim_{\tau \rightarrow \infty} \left(\frac{p}{R} \right)^\tau V_{jt+\tau}. \quad (2)$$

We define investment payoff as $\pi_{jt} \equiv -k_{j\tau+1} + \frac{p}{R} \left(\varphi_j k_{j\tau+1}^\alpha - c^f \right)$. Potential entrants with $V_{jt} \geq 0$ choose to enter the market, labeled as *entrants*.

Fundamental Component. The following equation is a popular assumption made in the literature to exclude bubbles:

$$\lim_{\tau \rightarrow \infty} \left(\frac{p}{R} \right)^\tau V_{jt+\tau} = 0, \quad (3)$$

which is called *the no-bubble condition*. This condition implies that the value of a firm is equal to the net present value of the investment payoff. Profit optimizing firms choose investment policy function $k_{jt+1} = k^*(\varphi_j)$ that satisfies the following first order condition: $\frac{p}{R} \alpha \varphi_j k^*(\varphi_j)^{\alpha-1} = 1$. The corresponding maximized investment payoff is: $\pi^*(\varphi_j) = \frac{p}{R} \left[(1 - \alpha) \left(\frac{\alpha p}{R} \right)^{\frac{\alpha}{1-\alpha}} \varphi_j^{\frac{1}{1-\alpha}} - c^f \right]$. The associated output is $y^*(\varphi_j) = \left(\frac{\alpha p}{R} \right)^{\frac{\alpha}{1-\alpha}} \varphi_j^{\frac{1}{1-\alpha}}$. Therefore, under the no-bubble condition, a unique solution exists for the value of firms:

$$V_{jt} = F_{jt} \equiv \frac{R}{R-p} \pi_{jt} = \frac{p}{R-p} \left[(1 - \alpha) \left(\frac{\alpha p}{R} \right)^{\frac{\alpha}{1-\alpha}} \varphi_j^{\frac{1}{1-\alpha}} - c^f \right], \quad (4)$$

where F denotes a firm's value under the no-bubble condition. We label F the fundamental value of firms.

Bubble Component. Other solutions to Equation (1) exist, insofar as we relax the no-bubble condition (3). Consider the following solution:

$$V_{jt} = F_j + B_{jt}, \quad (5)$$

where F_j is defined in (4) and B_{jt} follows:

$$B_{jt} = \begin{cases} B_0, & \text{Entrants} \\ \frac{R}{p} B_{jt-1}, & \text{Incumbents} \end{cases} \quad (6)$$

where $B_0 \geq 0$. The process (6) satisfies $B_{jt} = \frac{p}{R} B_{jt+1}$. It is straightforward to verify that Equations (5) and (6) indeed solve Equation (1). In Equation (5), the value of a firm is decomposed into two components. The first component F represents the net present value of investment payoffs, as defined in Equation (4). The second component B_{jt} is a pyramid scheme. Hereafter, it is labeled as a bubble. Free-disposal required that $B_0 \geq 0$. Note that Equation (5) nests Equation (4) as a special case when $B_{jt} = 0, \forall t, j$.

There exists a unique fundamental component F . However, since B_0 can take arbitrary values, infinitely many processes exist for B_t that are consistent with (6). Any solution satisfying Equations (5) and (6) also satisfies the conditions that risk-neutral firm owners have rational expectations and solve their problems optimally. Hence, bubbles (B_{jt}) are labeled as rational bubbles in the literature. To rational investors, as $B_{jt} = \frac{p}{R} B_{jt+1}$, the value of the bubble component B_{jt} is exclusively backed by the expectation of future bubble B_{jt+1} , instead of investment payoffs $\{\pi_{j\tau}\}_{\tau=t}^{\infty}$.

Online Appendix A.2 shows that $R < 1$ is necessary to ensure the existence of a steady-state equilibrium at which rational bubbles arise. We generalize this condition in an environment with a long-term growth trend in Section 3.4.

2.2. The Effects of Bubbles on the Aggregate Output and Productivity

Cutoff Firms. The bubble components attached to firms' values affect firms' entry decisions; therefore, bubbles affect the number of firms in the market. Figure 1 illustrates the two conditions that characterize the cutoff entrants. The upward sloping curve depicts the investment payoff function, i.e., $\pi = \frac{p}{R} \left[(1 - \alpha) \left(\frac{\alpha p}{R} \right)^{\frac{\alpha}{1-\alpha}} \varphi_j^{\frac{1}{1-\alpha}} - c^f \right]$. The horizontal line describes the reservation investment payoff under the Free Entry condition (FE): the value of a cutoff entrant ($\underline{V}$) is equal to zero

$$\underline{V} = \frac{R}{R - p} \pi + B_0 = 0. \quad (7)$$

Without a bubble component ($B_0 = 0$), Equation (7) implies that $\underline{\pi}^{NB} = 0$. The associated cutoff productivity is $\underline{\varphi}^{NB}$. A potential entrant enters the market if and only if its productivity is higher than $\underline{\varphi}^{NB}$.

In the presence of bubbles ($B_0 > 0$), the FE condition (the *dashed* line in Figure 1) is

given by $\frac{R}{R-p}\underline{\pi}^B + B_0 = 0$, or equivalently, $\underline{\pi}^B = -\frac{R-p}{R}B_0$. Bubbles shift the FE downward and subsidize firm entry: cutoff entrants accept a lower investment payoff. Consequently, the equilibrium cutoff productivity level is $\underline{\varphi}^B$, which is smaller than $\underline{\varphi}^{NB}$. That is, bubbles lead to the entry of the firms that would not enter without bubbles.

Moreover, the average productivity at the steady state, $\tilde{\varphi}_{ss} \equiv \frac{\int_{\underline{\varphi}}^{\infty} \varphi d\Phi(\varphi)}{\int_{\underline{\varphi}}^{\infty} d\Phi(\varphi)}$, is smaller in the presence of bubbles since bubbles induce the entry of less productive firms.³

Aggregate Output. At the steady state, the aggregate output is given by

$$Y_{ss} \equiv \int y_j dj = \frac{p}{1-p} \int_{\underline{\varphi}}^{\infty} y^*(\varphi) d\Phi(\varphi) = \frac{p}{1-p} \left(\frac{\alpha p}{R}\right)^{\frac{\alpha}{1-\alpha}} \int_{\underline{\varphi}}^{\infty} \varphi^{\frac{1}{1-\alpha}} d\Phi(\varphi), \quad (8)$$

where $\frac{p}{1-p} \int_{\underline{\varphi}}^{\infty} d\Phi(\varphi) = \sum_{\tau=1}^{\infty} p^{\tau} \int_{\underline{\varphi}}^{\infty} d\Phi(\varphi)$ denotes the cumulative distribution function of φ at the steady state. It can be inferred that bubbles stimulate the aggregate production by reducing $\underline{\varphi}$. Intuitively, firms that would produce without bubbles still produce the same amount in the presence of bubbles. In addition, more firms join the market and increase the aggregate output along the extensive margin.

Aggregate Capital and Productivity. The aggregate output can be written in the form of the aggregate production function

$$Y_{ss} = \Psi_{ss} K_{ss}^{\alpha}, \quad (9)$$

where $K_{ss} = \frac{p}{1-p} \left(\frac{\alpha p}{R}\right)^{\frac{1}{1-\alpha}} \int_{\underline{\varphi}}^{\infty} \varphi^{\frac{1}{1-\alpha}} d\Phi(\varphi)$ denote the aggregate capital stock and Ψ_{ss} represents the Solow residual, or the aggregate productivity at the steady state. We have

$$\Psi_{ss} = \frac{Y_{ss}}{K_{ss}^{\alpha}} = \left(\frac{p}{1-p}\right)^{1-\alpha} \left[\int_{\underline{\varphi}}^{\infty} \varphi^{\frac{1}{1-\alpha}} d\Phi(\varphi) \right]^{1-\alpha}. \quad (10)$$

Bubbles stimulate both the aggregate capital stock and the aggregate productivity through lowering $\underline{\varphi}$. To better understand the effects of bubbles on *aggregate* productivity and *average* productivity, it is sensible to rewrite the production function as $y_j = (\varphi_j^{\frac{1}{1-\alpha}})^{1-\alpha} k_j^{\alpha}$. One can interpret $\varphi_j^{\frac{1}{1-\alpha}}$ as the intangible input and the modified production function displays a *constant returns to scale*. Moreover, let $\Psi_{ss}^{\frac{1}{1-\alpha}} = \left(\frac{p}{1-p}\right) \left[\int_{\underline{\varphi}}^{\infty} \varphi^{\frac{1}{1-\alpha}} d\Phi(\varphi) \right]$ denote the aggregate stock of the intangible inputs. The aggregate stock of intangible inputs increases in $\underline{\varphi}$: An economy with a smaller cutoff value is associated with more firms and, therefore,

³A more formal proof is provided in Online Appendix A.3.

more total/aggregate intangible inputs.

As is discussed above, bubbles induce the entry of less productive firms ($\varphi^B < \varphi^{NB}$). Therefore, the *average* productivity is smaller in the presence of bubbles. However, the *aggregate* productivity (which is proportional to the aggregate stock of the intangible inputs) increases in the size of aggregate bubbles (B_0). These results are due to the effects of bubbles along the extensive margin.

The rise in the intangible inputs raises the demand for capital, and lead to an increase in the aggregate capital stock, K_t , which further boosts the aggregate output.

Note that these results are not determined by the assumption that the supply of capital is perfectly elastic. In Online Appendix A.4, we provide a similar example with an inelastic input supply, where bubbles also stimulate the aggregate output and productivity.

These results are shared by the complete model presented in Sections 3 and 4. In this stylized example, bubbles do not affect fundamental components F . However, in the complete model, bubbles affect firms exit decisions and therefore, affect the fundamental component of firms' values.

3. Model

Our model builds on standard firm dynamics models with competitive markets. We extend the existing models (e.g., [Clementi and Palazzo 2016](#)) by introducing a firm-level bubble component. In the model, bubbles influence the firms' selection mechanism. As a firm's exit incurs the loss of bubbles, bubbles discourage incumbents from exiting. Moreover, prospective entrants are encouraged to enter the market to obtain bubbles.

3.1. Preferences

Time is discrete and the horizon is infinite. There is no aggregate uncertainty, and the economy is populated with overlapping generations. In each period, a new generation of individuals who live indefinitely joins the economy. A representative household i in the new generation maximizes lifetime utility: $U_{it} = E_{it} \sum_{\tau=0}^{\infty} \beta^\tau C_{it+\tau}$ where $\beta \in (0,1)$ is the discount factor and $C_{it} \geq 0$ denotes the consumption.

The size of a given cohort does not change after it is born. Moreover, the size of every cohort is $1 + g$ of the size of its predecessor. The new cohort's size N_t at t follows: $N_{t+1} = (1 + g) N_t$. Consequently, as the population is joined by cohorts of increasing size, the size of the entire population also grows at the rate g .

3.2. Firms

Firms produce a homogeneous good in a perfectly competitive market. The production technology features decreasing returns to scale: $y_{jt} = A_t \varphi_{jt} k_{jt}^\alpha$ with $\alpha \in (0,1)$. A_t

denotes the common productivity component that is identical across firms. φ_{jt} is an idiosyncratic productivity shock. k_{jt} denotes the capital stock, which is predetermined. The idiosyncratic productivity shock φ_{jt} follows a Markov process, $\log \varphi_{jt+1} = \rho \log \varphi_{jt} + \varepsilon_{jt+1}$, where $\rho \in (0, 1)$ and $\varepsilon_{jt} \sim N(0, \sigma^2) \forall t, \forall j$.

Firms are owned by households. A firm's equity is issued and traded in a frictionless market. For each firm, we normalize the number of shares to one. Firms operate indefinitely and are subject to endogenous exit. The start-of-period value of a firm is equal to

$$V(\lambda, \mu, k) = A\varphi k^\alpha - c^f + \max\{V^c(\lambda, \mu, k), V^x(k)\}, \quad (11)$$

where λ includes aggregate state variables, μ contains the idiosyncratic state variables other than capital stock k , and c^f denotes a fixed operation cost. We suppress the subscripts to reduce the notational burdens. A variable X' denotes the variable X in the next period. After production, the firms' owners decide whether or not to exit from the market by comparing their value of continuation V^c with the value of exit V^x .

If firms exit, they divest all the capital stock, and the owners will then receive the scrap value. Therefore, the value of exit is

$$V^x(k) = (1 - \delta)k - g(k, 0), \quad (12)$$

where δ denotes the depreciation rate during production and $g(k, k')$ denotes the cost of adjusting capital stock from k to k' . The adjustment cost follows a standard functional form:

$$g(k, k') = c_0 \mathbb{1}\{k' \neq (1 - \delta)k\} k + c_1 \left(\frac{k' - (1 - \delta)k}{k} \right)^2 k, \quad (13)$$

where $c_0, c_1 \in (0, 1)$, and $\mathbb{1}\{k' \neq (1 - \delta)k\}$ is the indicator function that equals 1 if firms invest and equals 0 otherwise.

Firms continue to operate and invest if and only if $V^c(\lambda, \mu, k) \geq V^x(k)$. If a firm remains in the market, the owners can trade the shares in a friction-less market and decide how much to invest for future production. The continuation value of a firm is equal to

$$V^c(\lambda, \mu, k) = \max_{k'} \left\{ -I(k, k') + \frac{1}{R} \int V(\lambda', \mu', k') dJ(\lambda', \mu' | \lambda, \mu) \right\}, \quad (14)$$

where $I(k, k') \equiv k' - (1 - \delta)k + g(k, k')$, $J(\lambda', \mu' | \lambda, \mu)$ denotes the transition probability of

λ and μ , and R denotes the return on firm equity.⁴ The factors that the vector μ includes depend on whether the no-bubble condition is imposed.

The Fundamental Component. Online Appendix A.5 shows that

$$V^c(\lambda, \mu, k) = F^c(\lambda, \mu, k) + B, \quad (15)$$

Equation (15) is analogous to Equation (2): $F^c(\lambda, \mu, k)$ is the net present value of expected cash inflows $\pi(\lambda, \mu, k)$

$$\pi(\lambda, \mu, k) \equiv -I(k, k') + \frac{1}{R} \int_{\Psi'} V^x(k') dJ(\lambda', \mu' | \lambda, \mu) + \frac{1}{R} \int \left[A' \varphi'(k')^\alpha - c^f \right] dJ(\lambda', \mu' | \lambda, \mu),$$

where $\Psi' \equiv \{(\lambda', \mu') | V^c(\lambda', \mu', k') < V^x(k')\}$, and k' follows the optimal decision rule. We label $F^c(\lambda, \mu, k)$ as the fundamental component of the continuation value $V^c(\lambda, \mu, k)$. We label the difference between the continuation value and its fundamental component, B , bubble component.

A popular assumption to exclude bubbles in the literature is to impose $B = 0$, which is also called the no-bubble condition. Under this assumption, a firm's value solely consists of its fundamental component. In this scenario, the idiosyncratic state μ consists of the idiosyncratic productivity shock φ . We relax this assumption in the study. Consequently, the bubble component of the $V^c(\lambda, \mu, k)$ arises, and the firm-level bubble component emerges as a new idiosyncratic state variable.

The Bubble Component. Hereafter, the no-bubble condition is relaxed. Following the studies on the rational bubbles, this paper models B as a pyramid scheme:

$$B = \frac{1}{R} \int_{\Phi'} B' dJ((\lambda', \mu' | \lambda, \mu)), \quad (16)$$

where $\Phi' \equiv \{(\lambda', \mu') | V^c(\lambda', \mu', k') \geq V^x(k')\}$. Online Appendix A.6 verifies that (16) is consistent with (15). Only the bubbles of surviving firms are tradable. Bubbles can be considered as assets without intrinsic value: the assets that do not generate cash inflows. The size of a bubble depends solely on the expectation of its future size. This paper assumes free disposal of bubbles such that $B \geq 0$. The solution under the no bubble condition $V^c(\lambda, \mu, k) = F^c(\lambda, \mu, k)$ is a special case when $B = 0$ for all firms in each period.

⁴Households are risk-neutral, and therefore the equity market clears only if all firms have the same expected return.

We assume that bubbles are stochastic and subject to investor sentiment shocks. Bubble-less firms ($B = 0$) continue to be bubble-less in subsequent periods. Regarding a bubbly firm ($B > 0$) that does not exit the market, its bubble component can collapse (hit by a negative sentiment shock) with a probability $1 - p^b$. With probability p^b , the bubble component rolls over. Firms are subject to the negative sentiment shock with a common probability. However, the realization of the bubble crashes is independent across firms. Among those firms whose bubble components carry over to the next period, we assume that their bubble components evolve according to $B' = \left(\frac{1}{R} \cdot p^b \cdot p^o(\lambda, \mu, k')\right)^{-1} B$; thus, the process that governs the dynamics of the bubble component is

$$B' = \begin{cases} 0, & \text{with probability } 1 - p^b \\ \left(\frac{1}{R} \cdot p^b \cdot p^o(\lambda, \mu, k')\right)^{-1} B, & \text{with probability } p^b \end{cases} \quad (17)$$

where $p^o(\lambda, \mu, k') \equiv \int_{\Phi'} dJ^o(\lambda', \mu' | \lambda, \mu)$. $J^o(\lambda', \mu' | \lambda, \mu)$ denotes the transition probability of λ and μ conditional on the roll-over of the bubble component. $p^o(\lambda, \mu, k')$ is the survival probability if the firm does not face a bubble crash in the next period.

The discounted value of the next period's bubble is equal to

$$\frac{1}{R} \int_{\Phi'} B' dJ((\lambda', \mu' | \lambda, \mu)) = p^b \cdot \frac{1}{R} \int_{\Phi'} \left(\frac{1}{R} \cdot p^b \cdot p^o(\lambda, \mu, k')\right)^{-1} B dJ^o(\lambda', \mu' | \lambda, \mu) = B.$$

Hence, Equation (17) guarantees Equation (16) to hold. The process (17) ensures that the bubbles are consistent with investor optimality and rational expectations while maintaining the model's tractability. As in Galí (2014) and Martin and Ventura (2016), bubbles follow a “backward-looking” process so that the size of future bubbles (if bubble crashes do not occur) can be specified, given the current bubble size. The “backward” feature of the process facilitates our analysis because we can record the dynamics of bubbles on a given route.⁵

A key feature of (17) is the idiosyncratic bubble crashes. Hence, our model allows for the coexistence of bubbly and bubble-less firms. Equation (17) also implies that the growth rate of bubbles is increasing in the probability of a bubble crash and firm exit. Intuitively, bubbles must grow fast enough to compensate for the possible loss if firms are likely to exit or if bubbles are likely to crash. Alternative processes of bubbles con-

⁵In contrast, Farhi and Tirole (2012), Miao and Wang (2018) characterize the size of bubbles through forward-looking solutions.

sistent with investor rationality exist. However, these alternative processes do not affect the existence of bubbles' effects along the extensive margin and the selection effects of bubbles emphasized in this study. Our approach is a simple method for incorporating idiosyncratic bubble crashes.⁶

Discussions. We now discuss three features of the model. First, by relaxing the no-bubble condition, idiosyncratic states μ of a firm include the size of its bubble B , in addition to idiosyncratic productivity φ . To confirm this feature, recall that firms exit if $V^c(\lambda, \mu, k) < V^x(k)$. Bubbles enhance the value of continuation and reduce the probability of firms exiting.

Note that the model could be solved by imposing/assuming that the policy function and value functions do not depend on B , as done conventionally in the literature of firm dynamics. Such a solution is valid. However, this is equivalent to imposing the no-bubble condition.⁷ This study relaxes the no-bubble condition and focuses on the case with $B > 0$.

Second, studies on rational bubbles focus on discrete aggregate bubble shocks: shocks that shift the entire economy between a bubbly state and a bubble-less state.⁸ In our model, bubble shocks can be captured by continuous changes in p^b . A decrease in p^b implies a lower likelihood to roll over the bubble component. If p^b drops to 0, the entire economy shifts to a bubble-less state. In Section 4, we study the transition dynamics (perfect-foresight equilibrium paths) following unexpected changes in p^b .

Finally, bubbles in our model can affect the real economic activities only through their effects on firm entry and exit. Bubble components would have no real effects if one shut down entry and exit in the model.

3.3. New Cohorts and Firm Entry

Only newborn households have the opportunities to create new firms. We assume that each individual of the new generation has access to a unit continuum of projects. Projects can become firms that eventually produce. These projects are called *potential entrants*. The problems of potential entrants are similar to that of incumbents: they choose whether to produce in the following period, that is, to enter the market or not. If they decide to produce, they choose the amount of investment. Each potential entrant has its own realization of productivity and bubble. A potential entrant's productivity is drawn from

⁶As stated by Galí (2014) and Martin and Ventura (2018), how to choose the process is still debated.

⁷For example, recall that in Section 2, if V is expressed as a function of φ , there exists a unique solution to Equation (1), the fundamental component $F(\varphi)$. However, if V is expressed as a function of φ and B_t , then $V_t = V(\varphi, B_t) = F(\varphi) + B_t$ with $B_t = \frac{1}{R} B_{t+1}$.

⁸See, e.g., Martin and Ventura (2012) and Miao and Wang (2018).

the distribution $\varphi_t \sim \log N\left(-m \frac{\sigma}{\sqrt{1-\rho^2}}, \frac{\sigma^2}{1-\rho^2}\right)$, where m captures the possible difference between the initial productivity draw and the idiosyncratic productivity process.⁹ We assume that the investors are optimistic about a potential entrant with probability p^b —the same parameter that governs the survival of existing bubbles: an increase in p^b not only renders incumbents more possibly to remain bubbly, but also makes potential entrants more likely to become bubbly. If investors are optimistic, the potential entrants receive an initial bubble B_0 before their entry.

Analogous to incumbents, potential entrants can only issue tradable equity if they decide to produce in the subsequent period. Potential entrants enter the market if and only if the value of entry is non-negative: $V^e(\lambda, \mu) \geq 0$. Potential entrants start with zero capital stock. Hence, the value of entry is:

$$V^e(\lambda, \mu) = V^c(\lambda, \mu, 0). \quad (18)$$

Following (15), the value of entry can be decomposed into the sum of a fundamental and bubble component. The latter is equal to B_0 or 0 depending on the realization of market sentiment. We define *entrants* as the potential entrants who decide to enter.

3.4. Bubbly Equilibrium

A dynamic equilibrium consists of value functions, decision rules, prices, and the measure of firms, which satisfy the optimality conditions of firms and households, and clear all markets. Besides, in equilibrium, the measure (number) of firms' law of motion is consistent with the decision rules of investment, entry, and exit. Online Appendix A.7 introduces a formal definition of equilibrium.

One can characterize the equilibrium path for a given pair of p^b and B_0 , which are exogenously determined. Online Appendix B describes the solution method to obtain the equilibrium path. Section 4 calibrates the BGP and studies the transition dynamics. Along the BGP, the relative composition of firms remains stable, while the total amount of firms increases at rate g .

R along a BGP. Along a BGP, the equilibrium investment return on equity is given by $R = \beta^{-1}$. R cannot be smaller than β^{-1} , otherwise the equity market cannot clear because no one holds shares. To better understand the result, consider an off-equilibrium scenario when $R < \beta^{-1}$. In this scenario, every one would sell off firms, depressing the share

⁹Although potential entrants draw productivity, they do not produce prior to entry. One may label the initial draw as the advance information about future productivity upon entering the market.

price, until the investment return is restored to $R = \beta^{-1}$. Moreover, R cannot exceed β^{-1} , otherwise no one ever consumes and the transversality condition is violated. Online Appendix A.8 provides the mathematical proof.

The Existence of a Bubbly BGP. Next, we discuss the condition under which a bubbly BGP is sustainable. At the firm-level, no upper bound exists for the size of bubbles, as indicated by Equation (17). However, the existence of a bubbly BGP requires that the aggregate bubble must be bounded relative to the aggregate output. Otherwise, the economy would eventually deplete its resources. Thus, we have the following proposition.

Proposition 1. *There exists a bubbly BGP only if $\beta > \frac{1}{1+g}$.*

Proof. See Online Appendix A.9. □

Online Appendix A.9 shows that the entry and exit feature of the model does not affect the growth rate of the aggregate bubble from continuing incumbents, which is equal to R . Endogenous exit affects the growth rate of bubbles at the micro-level. Specifically, due to the possibility of firm exit, rational expectation requires firm-level bubbles to grow faster than R : see Equation (17). At the aggregate level, owing to idiosyncratic bubble crashes associated with firm exits, the aggregate bubble from continuing incumbents grows at the same rate as the expected investment return, R . A BGP is sustainable insofar as the output growth rate exceeds the expected investment return.

Proposition 1 provides a necessary condition for rational bubbles to exist along a BGP. The existence of the BGP and transition paths are verified numerically for the quantitative model. Specifically, we verify that the computed equilibrium paths satisfy the definition in Online Appendix A.7, and the relative distribution of firms converges to a unique stationary distribution for given parameters.

Dynamic Inefficiency. This paper focuses on a bubbly economy where $\beta > \frac{1}{1+g}$. *Dynamic inefficiency* is a concept that is closely related to the bubble existence condition (Proposition 1). An economy is dynamically inefficient if it is possible to make one generation better off without making another generation worse off.

Online Appendix A.10 demonstrates that the BGP is dynamically inefficient and it is always possible to make a Pareto improvement by persistently transferring consumption from young generations to old generations. As is well known, bubbles are sustainable and potentially welfare-improving in a dynamically inefficient economy.¹⁰

¹⁰Samuelson (1958) and Tirole (1985) are two pioneers formalizing this idea.

Welfare. We define social welfare as the sum of the lifetime utility of all living households $U_t \equiv \int U_{it} di$, where $U_{it} = E_{it} \sum_{\tau=0}^{\infty} \beta^\tau C_{it+\tau}$. We have the following proposition

Proposition 2. *The social welfare at time t is given by*

$$U_t = \int V(\lambda_t, \mu_t, k_t) d\eta_t^p(\mu_t, k_t) + \int V^e(\lambda_t, \mu_t) d\chi_t(\mu_t),$$

where η_t^p denotes the cumulative measure of producing firms, and χ_t denotes the cumulative measure of entrants.

Proof. See Online Appendix A.11. □

The social welfare at time t is equal to the aggregate wealth held by living individuals at time t ; note that the wealth contains bubbles. An increase in bubbles would improve social welfare as long as it raises the aggregate wealth.

Online Appendix A.1 discusses the model's assumptions.

4. Results

Calibration. This paper calibrates the model on a BGP. Table 1 presents the parameter values. The model is in annual frequency. Panel A lists the pre-assigned parameters. This paper sets the discount factor $\beta = 0.98$, the decreasing returns to scale $\alpha = 0.65$, and the capital depreciation rate $\delta = 0.1$. The processes of the idiosyncratic productivity shocks are calibrated to the estimates provided by İmrohoroglu and Tüzel (2014): $\rho = 0.7$ and $\sigma = 0.38$. The growth rate g is set to 2.81%, which corresponds to the average growth rate of real GDP in the United States between 1976 and 2019.

Panel B reports the remaining parameters calibrated to minimize the distance between the selected targets and their model-implied counterparts. First, using data from the Business Dynamics Statistics, this paper targets the share of new establishments, and those of age 4. These two moments are related to business dynamism and firm life cycles.¹¹ Second, this paper targets PE ratios and consider two groups of firms in Compustat: firms above the 75th percentile of assets and those below. We treat the groups as two portfolios and calculate their average PE ratio between 1976 and 2016.¹² The model matches the two moments with the corresponding PE ratios for the firms that have survived at least

¹¹We calculate the shares of tradable firms, i.e., the firms that exist at the end of a period.

¹²Firms' earnings can be negative in the data, which distorts the ranking of the PE ratio as a proxy for the degree of over-value of a firm. In the calculation of the two PE ratios, observations with negative earnings are excluded. In the calibration of the model, the same filter is applied.

20 years. Finally, this paper targets investment-related moments, including the average investment rate, inaction rate, and dispersion of investment rate, as reported by [Zwick and Mahon \(2017\)](#).¹³ Table 2 reports the targeted moments in the data and the counterparts generated in the model. Overall, the model fits the selected moments well, except that the calibrated model underpredicts the PE ratio of big firms. However, the model matches the qualitative pattern in the data that the PE ratio decreases in size.

4.1. Firms' Life Cycles

Figure 2 plots the evolution of the exit rate, productivity, and capital of an average firm by age. We plot the life cycles of an average bubbly and bubble-less firms. Bubbly firms refer to those firms whose asset prices exceed their fundamental values. For both groups of firms, their capital and productivity increase in age. Therefore, older firms are less likely to exit.

Within an age cohort, an average bubbly firm features a lower exit rate, less capital, and a lower productivity across the entire life cycle than the respective variables of an average bubble-less firm. These results reflect that bubbles encourage the entry of the firms that otherwise would not enter the market without bubbles. Furthermore, with an asset bubble, firms have less incentive to exit. Consequently, bubbly firms are, on average, less productive and they accumulate less capital over the life cycle.

Online Appendix C.1 shows that in the model, conditional on idiosyncratic states φ and k , bubbles barely affect the investment decision. Therefore, the difference in capital stock between bubbly and bubbleless firms is explained by the selection effect that changes the productivity distribution of firms rather than the effects along the intensive margin.

4.2. Impulse Responses Functions

We now analyze the impacts of a bubble shock. To this end, we feed in an unexpected innovation to p^b at $t = 0$, and the innovation decays at the rate $\rho_p = 0.5$.¹⁴ The economy is assumed to return to its steady state (BGP) at $t = 100$. The IRFs are then computed under the perfect foresight assumption, as the deviation from the BGP.

Figure 3 plots the IRFs of selected variables. First, the positive bubble shock raises the aggregate equity value; the latter is equal to the social welfare according to Proposition 2.

¹³These moments can be found in [Zwick and Mahon \(2017\)](#)'s Appendix, Table B.1, Balanced Sample. To mimic their balanced sample, model moments are computed from firms that are alive for at least 12 years (the time dimension of their balance panel data) in the simulated panel data.

¹⁴Results are robust for using alternative values of $\rho_p \in [0, 1)$.

Second, the positive bubble shock stimulates the aggregate output, the aggregate capital stock, and the aggregate TFP (in natural logarithm: $tfp = y_t - \alpha k_t$).

As demonstrated in Section 2.2, firms that otherwise would exit or not enter the market without a bubble shock contribute positively to the aggregate stock of intangible input, which is proportional to the aggregate TFP. The rise in the intangible input also stimulates physical capital investment and ultimately raises the aggregate output.

Bubble shocks affect real economic activities along the extensive margin. An increase in the total number of firms results from declining exit and increasing entry (Figure 3). In the presence of bubbles, firms have more incentive to enter the market and are less likely to exit. Interestingly, the entry rate shows an overshooting pattern because, in response to an expansionary bubble shock, there is a faster increase in the number of entrants than in the total number of firms in the short run but slower in the medium-run.

Finally, the bubble shock shifts the distribution of firms. Particularly, the average firm-level productivity declines. This result is due to the selection effect of bubbles: a bubble shock increases the fraction of bubbly firms that are less productive on average.

The IRFs are slow-moving and hump-shaped, and the effects of a bubble shock slowly build up even if the shock vanishes. Two reasons explain these features. First, the bubble shock increases the mass of less-productive firms because of the selection effect of bubbles. Less-productive firms grow faster than their more-productive counterparts in subsequent periods because of the mean-reverting feature of idiosyncratic productivity. Hence, increasing the mass of less-productive firms on impact precedes a further protracted increase in the aggregate production. Second, in the model, bubbles, once created, take time to vanish (a bubble collapses with probability p^b). As a result, through the extensive margin, bubble shocks exert slow *cumulative* effects on the number of firms over time. Online Appendix C.2 presents a decomposition exercise to study how firm entry and exit contribute to the aggregate output's slow-moving and hump-shaped impulse responses.

5. Empirical Evidence

Section 5.1 constructs a proxy for the fundamental component (and the corresponding non-fundamental component) of the asset prices. Section 5.2 discusses the identification strategy. Section 5.3 presents empirical findings.

5.1. A Proxy for the Fundamental Component

We study the portfolio of the top 500 firms (S&P 500) listed in the United States. The exact composition of this portfolio changes over time without affecting the representa-

tiveness of this asset. Let P_t denote the value of the portfolio that yields a stream of dividend (D_t). As discussed previously, the value (price) of such a portfolio can be decomposed into a fundamental component (F_t) and a bubble component (B_t), i.e., $P_t = F_t + B_t$. The fundamental component is the net present value of future dividends: $F_t \equiv E_t \left\{ \sum_{h=1}^{\infty} \left(\prod_{j=0}^{h-1} \Lambda_{t+j} \right) D_{t+h} \right\}$, where Λ_{t+j} is the discount factor. Log-linearizing the previous equation leads to

$$f_t = c + \sum_{h=0}^{\infty} \left(\frac{\Gamma}{R} \right)^h \left[\left(1 - \frac{\Gamma}{R} \right) E_t \{d_{t+h+1}\} - E_t \{r_{t+h}\} \right], \quad (19)$$

where a variable in lower case denotes its log value and r_t denotes $-\log(\Lambda_t)$. Γ and R denote the growth rate of dividend and $1/\Lambda_t$ in the steady state, respectively. This paper uses the price-fundamental differential component and the non-fundamental component interchangeably to describe the term: $p_t - f_t$.

Our identification of "bubble shocks" relies crucially on the measure of the price fundamental differential component; however, the fundamental component f_t is not directly observed in the data. Following [Galí and Gambetti \(2015\)](#), we construct a proxy of this unobserved component using VAR forecasts, and thus obtain a proxy for the corresponding price-fundamental differential component. We consider a VAR consisting of the following variables: TFP, real GDP (y_t), real dividend (d_t), real stock price (p_t), real interest rate (r_t) and the total number of firms (N_t). The choice of variables contained in our VAR is motivated by our theoretical model.

Let $Y_t \equiv [TFP_t, y_t, d_t, p_t, r_t, N_t]'$, the reduced form representation of our VAR model is $Y_t = B(L)Y_t + U_t$, where L is the lag operator and $B(L)$ is the matrix of lag order polynomials. Using the VAR's forecasts as the empirical proxies for $E_t \{d_{t+h}\}$ and $E_t \{r_{t+h}\}$, the construction of a proxy for the fundamental component according to (19) is straightforward. Figure 4 plots the decomposition of the S&P 500 based on the estimation of our VAR.¹⁵ Online Appendix F provides the details regarding the source and sample coverage of the data set used in this study. In Online Appendix G, we demonstrate that the *linear* VAR adopted in this paper is a good approximation of our *non-linear* model introduced in Section 3. Specifically, using the data simulated from the theoretical model discussed in Section 4, Online Appendix G provides a simulation exercise illustrating that the *proxy* for the fundamental component constructed based on the VAR mimics the *true fundamental*

¹⁵The estimated price-fundamental differential component captures the well-known dot-com (around 2000) and housing bubble episodes (around 2008). The Online Appendix D discusses and shows that the constructed price-fundamental differential component is highly correlated (0.91) with the recent estimate provided by [Hanson \(2022\)](#).

in the model.

5.2. The Identification of "Bubble Shocks"

This section constructs the "bubble shocks" using the medium-run restriction, as pioneered by Uhlig (2003). In the literature, the medium-run restriction is widely used to identify news shocks (Barsky and Sims 2011).

The "bubble shocks" are identified as the shocks that maximize the forecast error variance decomposition (FEVD) of the constructed price-fundamental differential ($p_t - f_t$) in the subsequent H periods, controlling for productivity shocks (unanticipated and anticipated shocks in the near future) in the baseline; selected structural shocks such as risk premium shocks, and credit supply shocks, and shocks to the short-term interest rate in robustness checks. The identification strategy assumes that: fluctuations in our proxy of the non-fundamental component are mainly driven by bubble shocks; fundamental shocks, particularly the ones that are *not controlled* for in the VAR, have limited impacts on the proxy of the non-fundamental component. Online Appendix E provides in-depth discussions of the identification issue and the construction of the "bubble shocks".

We control for anticipated productivity shocks (news shocks) up to 3 years ahead, and H is set to be 40. The number of lags is chosen to be 4. $\frac{\Gamma}{R}$ is calibrated to 0.98 in the empirical analysis. Results are robust to alternative calibrations of $\frac{\Gamma}{R}$.

5.3. Results

Figure 5 plots the identified "bubble shocks". The shaded areas indicate the five recessions occurring in the sample. Each recession is associated with a big negative "bubble shock". More specifically, negative "bubble shocks" have contributed to the collapse of the dot-com and housing bubbles that are observed in Figure 4.

Figure 6 reports the impacts of positive "bubble shocks" on the aggregate economy in our baseline. Following Kilian (1998), this paper constructs standard errors from 2000 bias-corrected bootstraps. The shaded areas indicate the 95% confidence bands. Two features should be highlighted from these IRFs. First, a positive "bubble shock" has expansionary effects: it has positive impacts on real asset price, real GDP, and the aggregate TFP. Second, while the "bubble shock" has peak effects on asset prices and its non-fundamental component on impact (within a year), its effects on real economic activities are slow-moving and hump-shaped. These results are consistent with the effects of bubble shocks in the model.

More importantly, the transmission of "bubble shocks" in the data along the extensive margin is relevant: a one standard deviation "bubble shock" increases the total number of firms by roughly 0.5%, persistently.

To better understand the effects of "bubble shocks" along the extensive margin, this paper estimates the responses of other measures of firm dynamics to "bubble shocks". Particularly, we include the number of new firms, the entry rate, the number of exiting firms, and the exit rate into the empirical analysis. To avoid the curse of dimensionality, one variable of firm dynamics is included at a time. The red lines and the shaded area in Figure 7 show how the "bubble shocks" affect other measures of firm dynamics. Each row reports the results estimated from a separate SVAR model. The first row in Figure 7 reports the IRFs of the selected variables estimated from the baseline VAR considered earlier, and these results have been discussed before. They are repeated in Figure 7 to facilitate comparisons. The last four rows report the results estimated from separate VAR models by replacing the total number of firms by the number of new firms, the entry rate, the number of exiting firms, and the exit rate, respectively. The "bubble shocks" are normalized to have unit positive effects on real GDP at the horizon 10. The real effects of "bubble shocks" on real GDP revealed in the baseline are robust to alternative specifications.

Figure 7 demonstrates that in a boom driven by our identified "bubble shocks", both the total number of firms and the number of new firms increase. While the number of new firms increases in the short run, the increase in the total number of firms is more persistent in the medium-run. Therefore, the entry rate increases in the short run and declines persistently in the medium-run. The exit rate decreases. The total number of exiting firms decreases in the short run, followed by a persistent increase in the medium-run. The last result is driven by the rise in the total number of firms: holding the exit rate constant, an increase in the total number of firms leads to a higher number of exiting firms.

Figure 7 plots the predictions of the model (blue lines) together with the empirical findings (red lines). We introduce bubble shocks in the model such that the response of the real GDP in the model matches those estimated in the data (left column, Figure 7). Then, we compare the IRFs of firm dynamics variables to a bubble shock in the model to the counterpart estimated in the data in response to the "bubble shock" (right column, Figure 7). The empirical IRFs are constructed with estimation uncertainty: confidence bounds. For comparison, we compute two sets of IRFs from the model. The blue dashed lines report the responses to a, relatively big, bubble shock that matches the upper-bound effects on real GDP revealed in the data. Similarly, the blue dash-dotted lines are constructed by shocking the model economy with a, relatively small, bubble shock that matches the lower-bound effects on real GDP revealed in the data.

The blue lines (model's predictions) in Figure 7 overlap with the red shaded area

(data). Specifically, the IRFs of firm dynamics variables to a bubble shock in the model qualitatively and quantitatively match the IRFs to the identified shock in the data.

Simulation Exercises that Connect the SVAR to the Model. Simulation exercises conducted in Online Appendix G demonstrate that applying the medium-run restriction to the data simulated using the model uncovers the true IRFs in Section 4.2. Those IRFs are comparable to their empirical counterparts in Figure 6; however, conducting the same exercise using the data simulated from the model without bubbles fails to produce the empirical findings presented in Figure 6.

Bubble shocks do not affect TFP on impact; however, through the extensive margin, the total number of firms increases persistently after a positive bubble shock resulting in delayed and prolonged responses of TFP and real GDP. These dynamics mimic the effects of news shocks, as shown in Barsky and Sims (2011) and Kurmann and Sims (2021). The latter is defined as shocks that have no impact effects on current TFP but significant effects on future TFP. Therefore, bubble shocks can be viewed as the source of empirically-identified news shocks. The following simulation exercise verifies this interpretation.

Barsky and Sims (2011) and Kurmann and Sims (2021) identify news shocks by applying the medium-run/max-share restriction: news shocks most explain fluctuations in *future TFP*.¹⁶ Applying this identification strategy to model simulated data identifies bubble shocks, as shown in Figure 8, which plots the IRFs to a bubble shock in the model, and the estimated IRFs (based on simulated data) to a shock identified according to the news shock identification strategy. The estimated IRFs capture the effects of bubble shocks. This finding suggests that, potentially, bubble shocks can be a micro-foundation for empirically-identified news shocks.

Robustness Checks and Additional Results. Online Appendix D.2 demonstrates that the baseline empirical findings are robust to alternative specifications that control for risk premium shocks, monetary policy shocks, and credit supply shocks, separately.

Online Appendix D.3 presents additional evidence that supports the relevance of the effects of bubbles along the intensive margin emphasized by, e.g., Farhi and Tirole (2012), Martin and Ventura (2012), Miao and Wang (2018), and Ikeda and Phan (2019). Moreover, we present evidence that the effects of bubbles along the extensive margin play a dominating role in the information sector during the dot-com boom burst episodes.

¹⁶The max-share restriction maximizes the share of contribution of a shock at *one specific* horizon in the future. The results presented in this section are robust to using the max-share restriction.

6. Conclusion

A heterogeneous firm model with endogenous entry and exit is generalized to allow for the existence of rational bubbles. The framework introduces a new transmission channel through which asset bubbles affect real economic activities. Bubbles have both micro- and macro-level implications. At the firm-level, this study shows the selection effect of bubbles: low-productivity firms enter/stay in the market due to the presence of bubbles. At the aggregate level, we emphasize the effects of bubbles along the extensive margin: an increase in the aggregate bubble enhances real economic activities by increasing the number of new firms and decreasing the exit rate. Basing on a SVAR model, we provide empirical evidence that supports the effects of bubbles along the extensive margin.

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Table 1: Model Parameters

Panel A: Fixed Parameters		
Parameter	Description	Value
α	Decreasing returns to scale	0.65
ρ	Idiosy. shock persistence	0.70
σ	Idiosy. shock volatility	0.38
δ	Depreciation rate	0.10
β	Discount factor	0.98
g	Growth rate	2.81%

Panel B: Calibrated Parameters		
Parameter	Description	Value
m	Mean shift of entrants' prod.	1.63
B_0	Initial bubble component	443.57
c_f	Fixed cost of production	15.57
c_0	Fixed adjustment cost	$8.60 \cdot 10^{-4}$
c_1	Variable adjustment cost	0.039
p^b	Surviving probability of a bubble	0.92

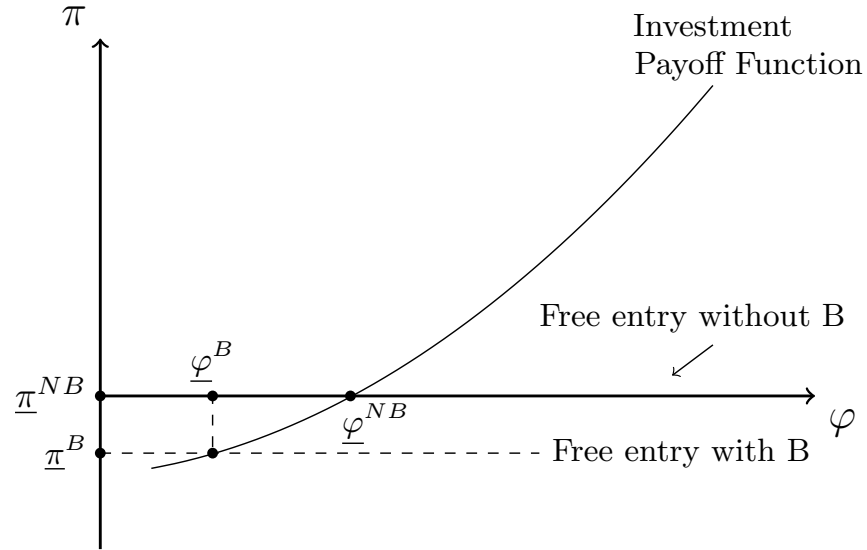
Note: This table reports the calibrated parameters.

Table 2: Calibration Targets and Model Fit

Moment	Data	Model
Share of new establishments	0.087	0.077
Share of four-year-old establishments	0.045	0.056
PE ratio of firms above 75th percentile of assets	16.36	8.50
PE ratio of firms below 75th percentile of assets	18.93	20.25
Investment inaction rate	0.237	0.220
Average investment rate	0.104	0.141
Standard deviation of investment rate	0.160	0.138

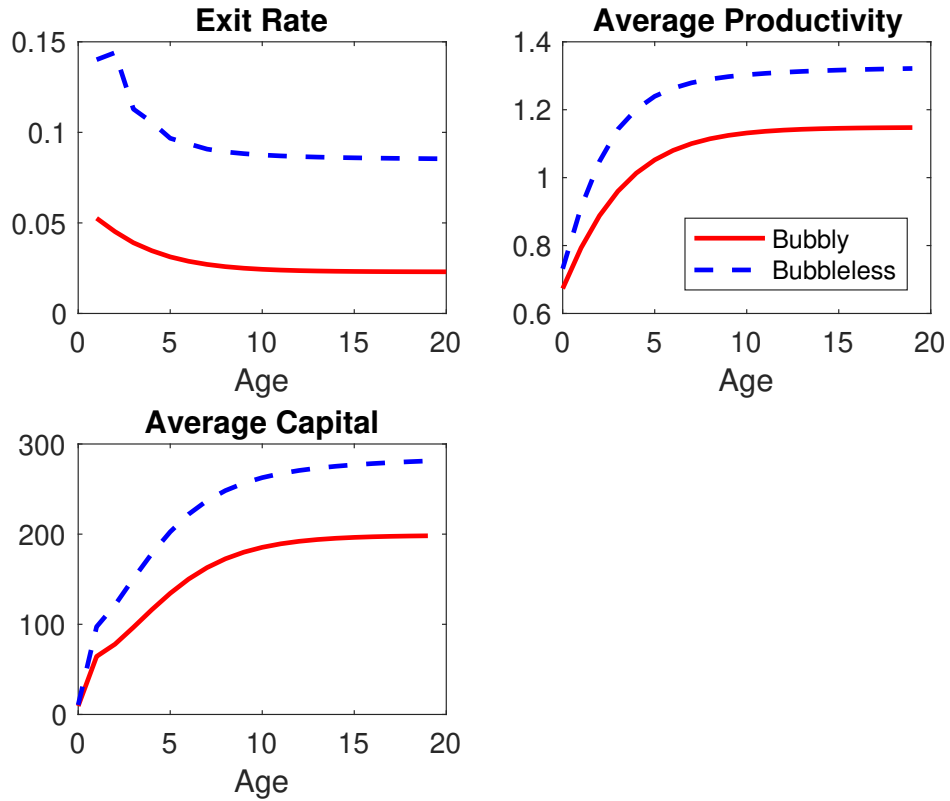
Note: This table reports the targeted moments used to calibrate the model and the model fit.

Figure 1: Bubbles and the Cutoff Productivity



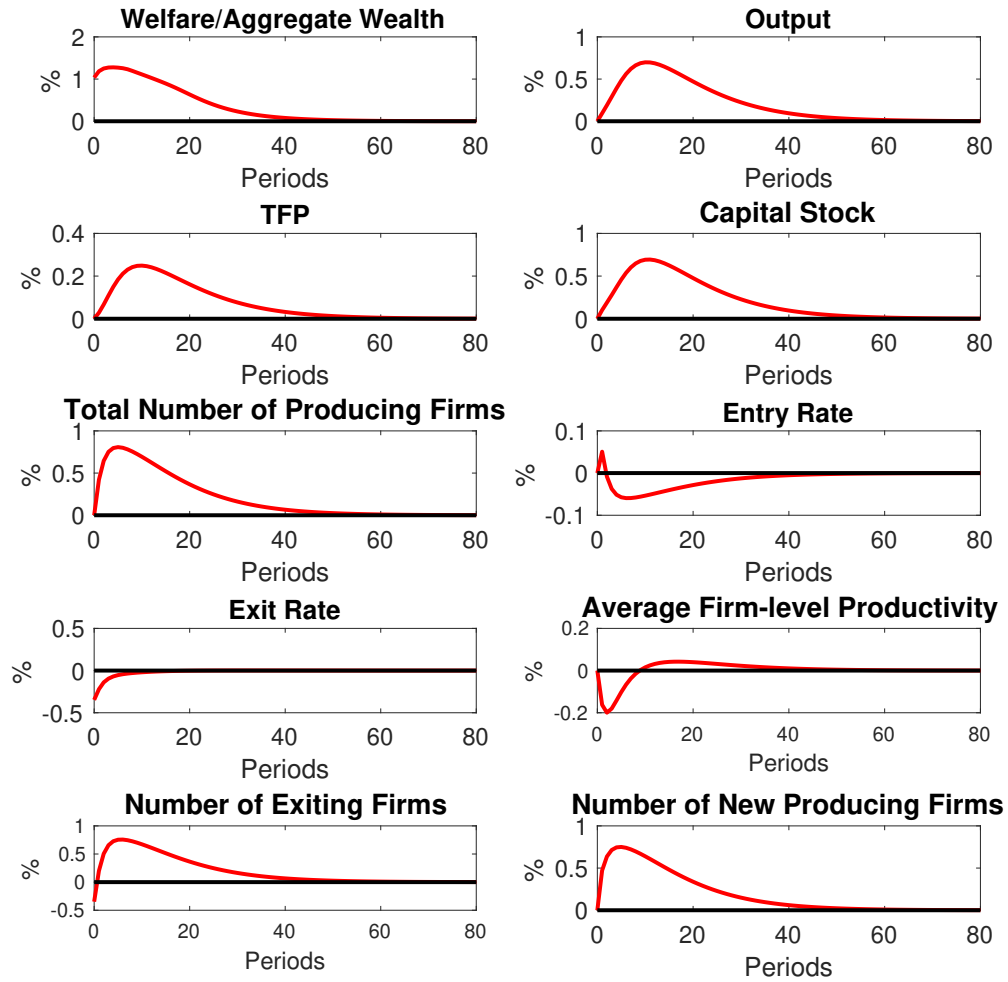
Note: This figure plots the investment payoff function and the free entry condition (with and without bubbles) that pin down the cutoff productivity.

Figure 2: Firms' Life Cycles



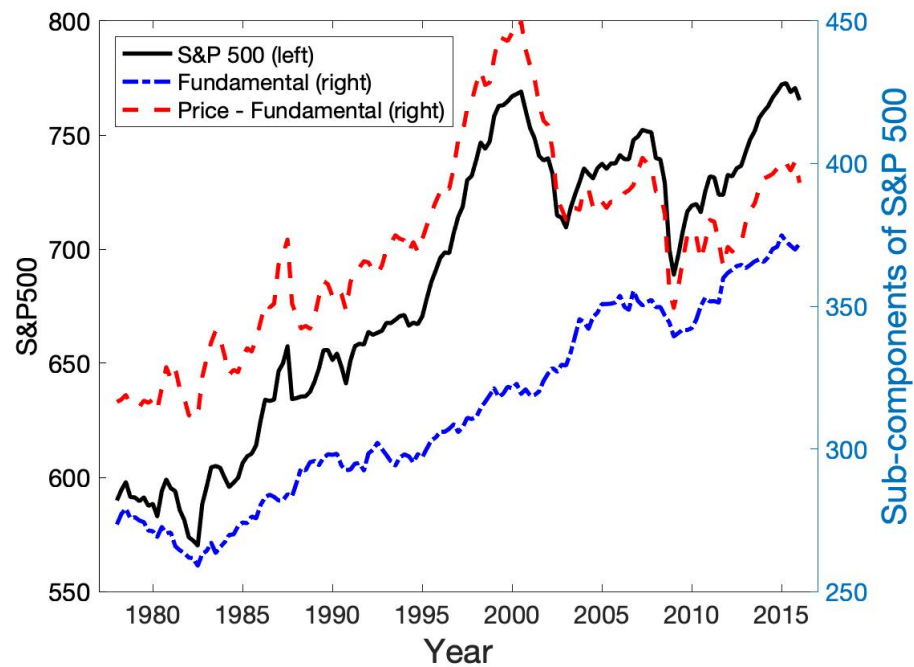
Note: This figure plots the evolution of the exit rate, productivity and capital of an average firm by age.

Figure 3: Impulse Responses to a Bubble Shock in the Model



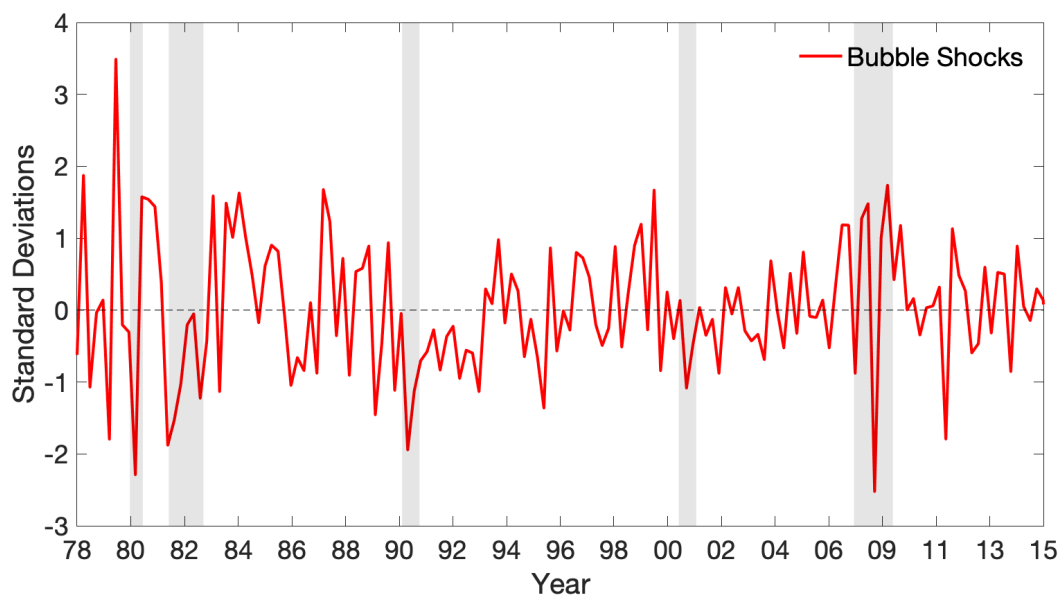
Note: This figure plots the IRFs to a positive bubble shocks in the model. The shock is normalized to increase the aggregate wealth on impact by 1%.

Figure 4: S&P 500 and its Decompositions



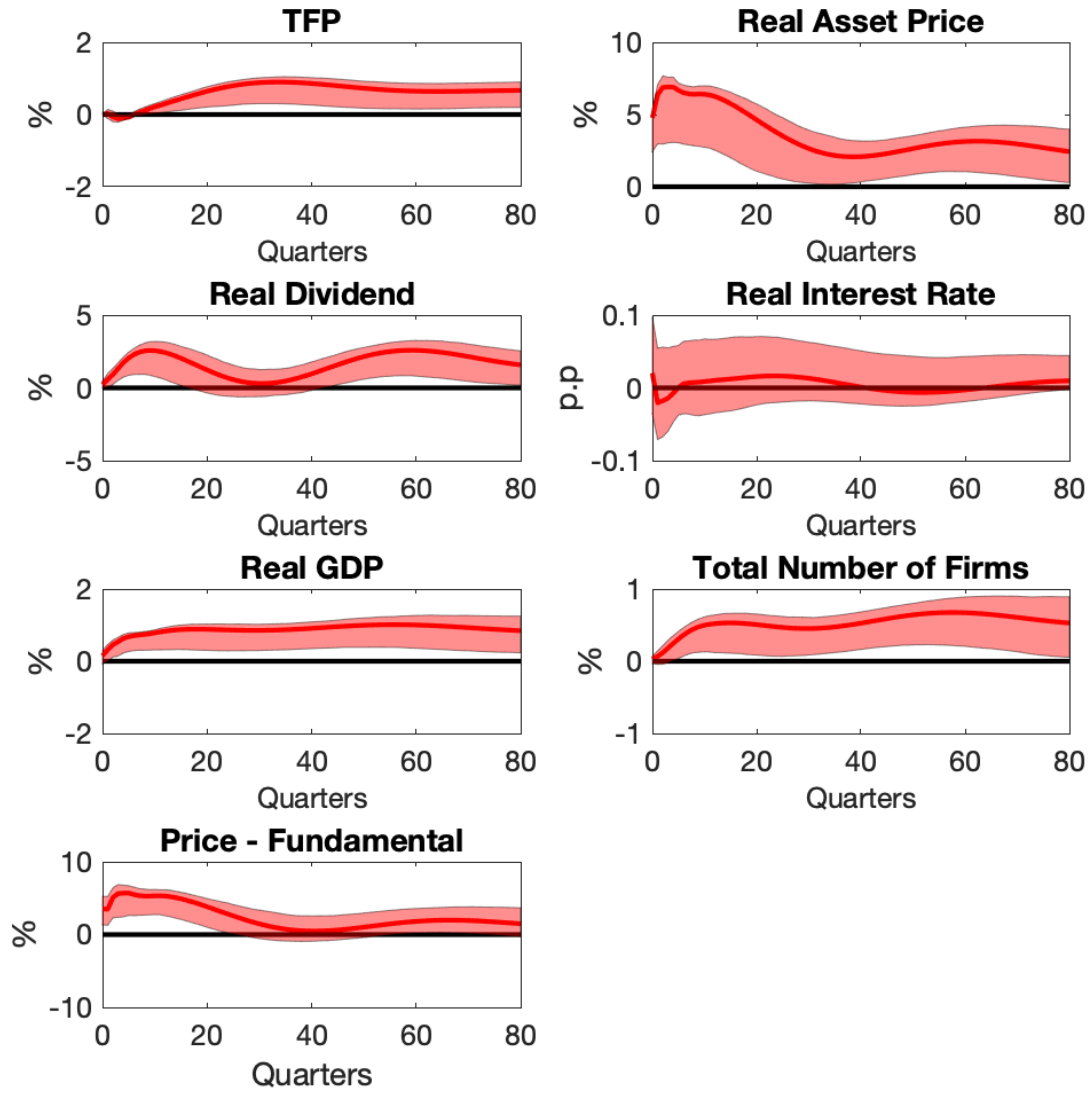
Note: This figure plots the evolution of the real value (in log) of Standard and Poors (S&P) 500 Index (solid black line with the y-axis on the left-hand side) together with its decomposition (with y-axis on the right-hand side): the fundamental component (dash-dot blue line) and the price-fundamental differential (dash red line). The fundamental component is constructed as the present value of future real dividends, where expected future dividends are constructed in real-time using forecasts based on a vector autoregressive model. Authors' own calculation.

Figure 5: Identified Bubble Shocks



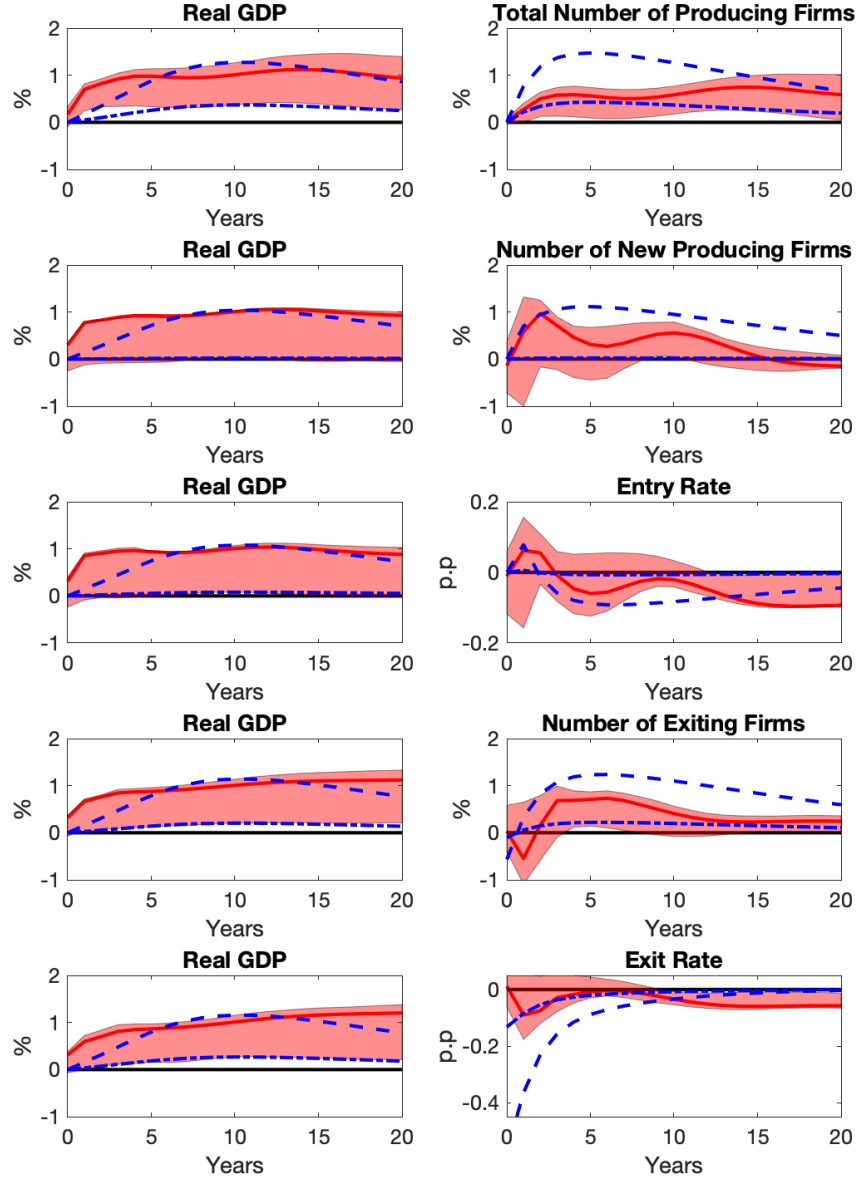
Note: This figure plots the identified "bubble shocks". The shaded area indicates the five recessions occurred in the sample.

Figure 6: Impulse Responses to a Bubble Shock in the Data



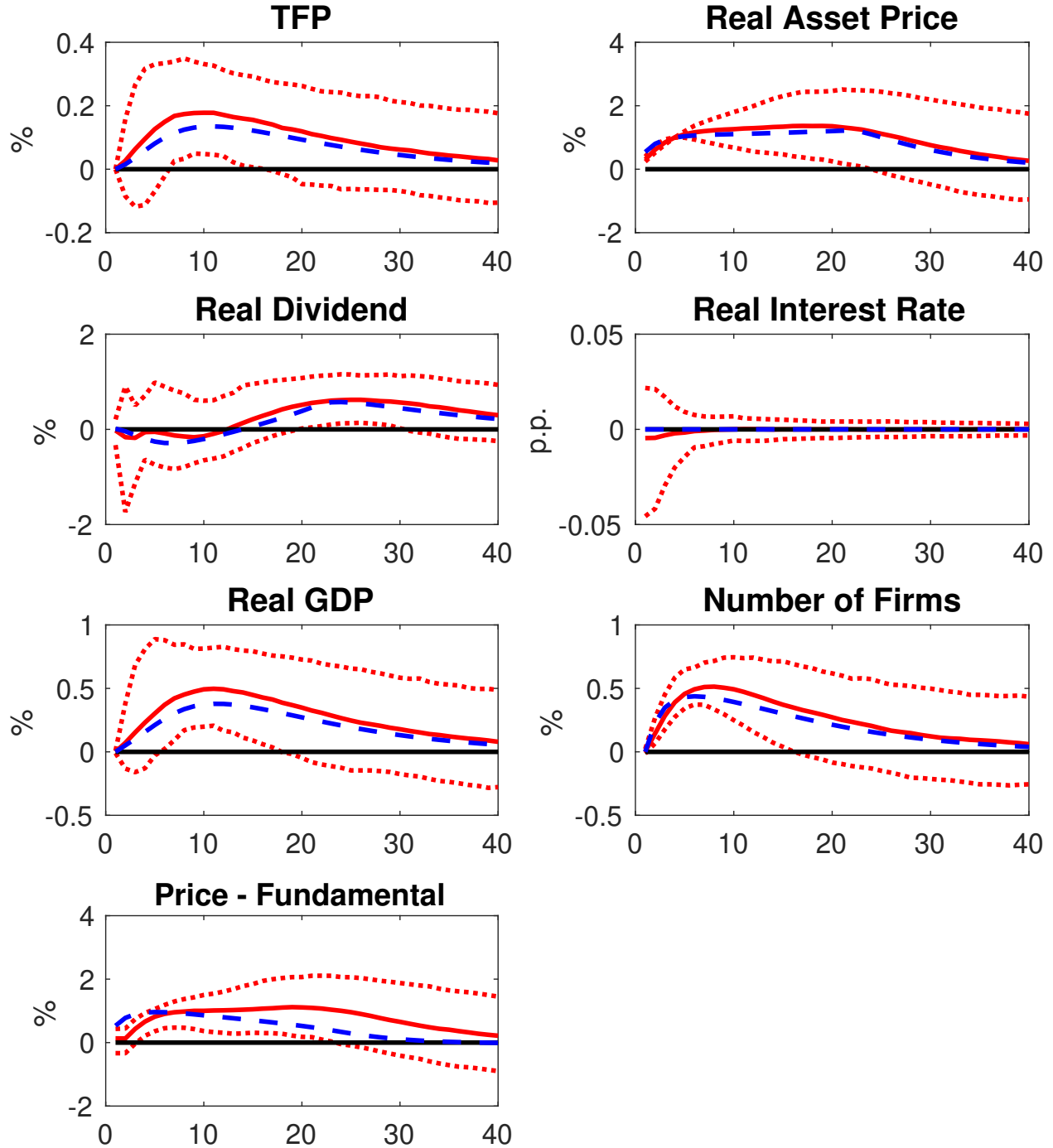
Note: This figure plots the impulse response functions to a one standard deviation "bubble shock" estimated based on the baseline SVAR. The shaded areas indicate the 95% confidence bands.

Figure 7: Impacts of a Bubble Shock on Firm Dynamics Variables



Note: This figure plots the IRFs to positive bubble shocks in the model (blue lines) together with the estimated IRFs from the data (red lines). The first row reports the IRFs estimated from the baseline VAR considered in Section 5.1. The last four rows report results estimated from separate SVARs, replacing the total number of firms by the number of new firms, the entry rate, the number of exiting firms, and the exit rate, respectively. The shaded areas indicate the 95% confidence bands. The blue dashed lines report the responses to a, relatively big, bubble shock that matches the upper-bound effects on real GDP revealed in the data. Similarly, the blue dash-dotted lines are constructed by shocking the model economy with a, relatively small, bubble shock that matches the lower-bound effects on real GDP revealed in the data.

Figure 8: Bubble Shocks and Empirically-identified News Shocks



Note: This figure plots the true IRFs to a positive bubble shocks in the model (blue dashed line) together with the estimated IRFs by applying news shocks' identification strategy to the simulated data (red lines). The solid red line indicates the median of the estimated IRFs from 1000 simulations. The dotted red lines represent the 90% intervals out of the 1000 simulations. Each simulated data set consists of 160 periods—the same sample length as in the data.