

Online Appendix for: Bubbly Firm Dynamics and Aggregate Fluctuations

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Appendix [A](#) is the Appendix for the model. Appendix [B](#) is the Appendix for numerical methods. Appendix [C](#) provides supplementary quantitative results. Appendix [D](#) presents additional empirical results and discussions. Appendix [E](#) provides in-depth discussions of the identification issue and construction of the "bubble shocks". Appendix [F](#) provides a brief overview of the data used in the paper. Appendix [G](#) applies the empirical strategies adopted in the paper using data simulated from our structural model.

A Appendix for the Model

A.1 Discussions about Model Assumptions

Public Listing In the model, we assume that the value of any firm, including new firms, can have a bubble component. This assumption seemingly contradicts the common narrative that only publicly listed firms might be traded with bubbles in the real world. We offer two arguments that defend the assumption.

First, when a non-publicly-listed firm is to be sold, its value can contain a rational bubble component as long as the buyer believes that the bubble component would roll over and future buyers would pay for this bubble component.

Second, conditional on the existence of rational bubbles, the common narrative that *only* publicly-listed firms have the bubble component is inconsistent with rational expectations. Consider a slightly more complicated version of the model: firms can become

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publicly listed once their capital stocks reach a cutoff level, and the process of IPO incurs zero cost. It would be a mistake to assume that bubble components can only be obtained when firms become listed. The reason is the following: Even if a firm does not reach the cutoff level of capital to be listed, as long as it has a positive probability to be listed in the future, rational expectations ensure that a positive bubble component is attached to the value of this firm.¹ Therefore, rational expectations imply that both listed firms and non-listed firms contain bubble components.

Firm Entry In our model, the opportunity to create new firms is only endowed to newborns. In fact, most of our qualitative and quantitative results remain unchanged if we relax this assumption and assume instead that all living individuals are endowed with the *same opportunity*: A unit continuum of potential entrants that draw initial productivity from the distribution $\varphi_t \sim \log N \left(-m \frac{\sigma}{\sqrt{1-\rho^2}}, \frac{\sigma^2}{1-\rho^2} \right)$, and receive the initial bubble component B_0 with probability p^b . The alternative assumption does not alter the relative distribution of *entrants*. Note that all the variables we study in Section 4 are determined by the relative distribution of firms, which is determined by the relative distribution of *entrants*.² Hence allowing all individuals to create firms (with same initial distribution) does not affect any of the results in Section 4.

Despite that most of our results are independent to the assumption that only newborns can create firms, there exists one exception: Proposition 2.³ When assuming all living individuals can create firms, the social welfare $U_t \equiv \int U_{it} di$, is still equal to the “wealth” of all households. However, since every individual can create firms in each period, the “wealth” does not only include the equity value of existing firms, *but also* includes the expected value of the non-existent firms to be created in the future: The social welfare is equal to the sum of the aggregate equity value of the existing firms and the discounted value of future firms. Overall, the result is still qualitatively similar to the result under the original assumption.

¹Note that the reasoning above does not require zero cost of IPO. As long as the bubble component to be obtained upon IPO is higher than the cost of IPO, firms always choose to be listed once they reach the threshold. Therefore, positive bubble components always exist on the firms that will possibly become listed in the future.

²More specifically, we present the law of motion for the relative distribution of firms at the end of A.7, in (23), (24) and (25), which imply that the relative distribution of firms remain unchanged given the alternative assumption.

³It can be easily inferred from A.9 that the alternative assumption does not affect Proposition 1.

Friction-less Markets One important distinction between our paper and other recent works of rational bubbles is our assumption of plain-vanilla competitive markets.⁴ The vast literature of macro-finance highlights the importance of financial frictions in the propagation of financial shocks. Previous studies of rational bubbles typically feature financial frictions as well.⁵ The literature argues that bubbles act as collateral or liquidity, and thus could relax financial constraints. One example of how this mechanism works is the following: When firms are subject to collateral constraint, bubbles can increase firms' collateral value and raise their borrowing capacity. Consequently, an increase in the aggregate bubble raises the aggregate output along the intensive margin.⁶

The framework in the current study shows that, even in the absence of market frictions, investor optimism can give rise to an asset price boom and an aggregate output expansion. This result rests on the mechanism that bubble components directly affect the entry and exit decisions of firms. The mechanism is a natural outcome after relaxing the no-bubble condition in an otherwise standard firm dynamics model. In fact, bubbles in our model can affect the real economic activities only through their effects on firm entry and exit. Bubble components would have no real effects if one substitutes our assumptions of endogenous firm entry and exit by assuming inelastic entry and exit, i.e., firms' entry and exit are independent to the continuation value.

Results presented in Section 4 can be viewed as a friction-less benchmark for the true impulse responses of an aggregate bubble shock in the real world. To incorporate frictional financial intermediaries is beyond the scope of this paper.⁷ As it is already uncovered in previous studies, in the presence of financial frictions, bubbles relax firms' financial constraints and thus, boost the real economic activities along the intensive margin. Therefore, financial frictions might further amplify the effects of bubbles on real economic activities.

Finally, note that financial frictions are also included in the models of rational bubbles

⁴Although our model is abstract from market frictions, it does feature a real friction: capital adjustment cost. We incorporate capital adjustment cost to better account for firm life cycles. It is worth noting that bubbles affect firm entry and exit regardless of the existence of capital adjustment cost.

⁵For example, see [Martin and Ventura \(2012\)](#), [Miao and Wang \(2018\)](#), [Ikeda and Phan \(2019\)](#). See, also, [Olivier \(2000\)](#) and [Queirós \(2019\)](#) who model asset bubbles in frictional goods markets.

⁶In a model with financial frictions, it is indeed questionable whether bubble shocks can be disentangled from other financial shocks, like the shocks studied by [Jermann and Quadrini \(2012\)](#), and [Gilchrist and Zakrajsek \(2012\)](#), since a shock to bubbles is also a shock to financial constraints. One may thus interpret bubble shocks as a way to model financial shocks in a frictional environment.

⁷In a companion paper, [Tang and Zhang \(2021\)](#), we develop a model of heterogeneous firms that have access to a frictional debt market, but no access to an equity market, and study the bubbles in the debt market.

to ensure $R < 1 + g$ —a well-documented phenomenon, and as implied in Section 3.4, a crucial condition for rational bubbles to exist. Examples include the works of [Farhi and Tirole \(2012\)](#), [Martin and Ventura \(2012\)](#), and [Ikeda and Phan \(2019\)](#).⁸ We follow a stylized way to introduce $R < 1 + g$. The lack of the store of value that provides dynamically efficient investment return ($R \geq 1 + g$) leaves open the possibility for welfare-improving bubbles—an idea that is originally formalized by [Samuelson \(1958\)](#).⁹

A.2 Sustainability of Bubbles in Section 2

For every firm, a rational bubble component grows over time. In aggregate, if bubbles persistently outgrow output, the world resource will ultimately become insufficient to purchase the bubbles. Under this condition, as argued by [Tirole \(1985\)](#), rational expectations exclude bubbles because investors expect bubbles to be unsustainable. Therefore, the existence of bubbles requires the aggregate bubble to remain stationary relative to the aggregate output along the equilibrium path.

This Appendix establishes the sustainability condition for a bubbly steady state. Owing to the entry and exit of firms, the aggregate bubble B_t^a evolves as follows:

$$B_{t+1}^a = p \cdot \frac{R}{p} B_t^a + M \cdot B_0 = R \cdot B_t^a + M \cdot B_0. \quad (1)$$

The right hand side of equation (1) comprises two parts: $R \cdot B_t^a$ represents the existing bubbles from the existing incumbent firms, and $M \cdot B_0$ represents the new bubbles brought by entrants, where M denotes the number of entrants. We assume that $R < 1$ such that B_t^a is stationary. Otherwise, bubbles are unsustainable. In fact, at the steady state: $B_t^a = \frac{MB_0}{1-R}$. It is straightforward to show that the total market capitalization of firms is also a constant. Specifically, bubbles are sustainable (i.e., the market for firm equity clears at the steady state) and consistent with rational investment behavior.

Because B_0 can take different values, multiple steady state equilibria exist. The realization of B_0 depends on market sentiment, or market psychology, as per [Asriyan et al.](#)

⁸An important contribution of this line of research is to show bubbles can arise in economies where investments are dynamically efficient. See [Abel et al. \(1989\)](#) and [Geerolf \(2018\)](#) for empirical discussions of this topic.

⁹Our approach of incorporating bubbles is similar to that of [Gali \(2020\)](#), who develops a model with overlapping generations of perpetual youth that survive indefinitely, but are subject to a constant probability of death and retirement. Note that in his model, bubbles can only exist if $\nu < \beta$, where ν represents the probability of remaining active and correspond to $\frac{1}{1+g}$ in Section 3 in our model.

(2020).

A.3 The Average Productivity

The average productivity $\tilde{\varphi}_t$ is a function of cutoff productivity $\underline{\varphi}$. Its derivative with respect to $\underline{\varphi}$ is equal to

$$\tilde{\varphi}'(\underline{\varphi}) = \frac{\Phi'(\underline{\varphi}) \left\{ \int_{\underline{\varphi}}^{\infty} \varphi d\Phi(\varphi) - \underline{\varphi} [1 - \Phi(\underline{\varphi})] \right\}}{\left[\int_{\underline{\varphi}}^{\infty} d\Phi(\varphi) \right]^2},$$

where the distribution density $\Phi'(\varphi) \geq 0$, $\int_{\underline{\varphi}}^{\infty} \varphi d\Phi(\varphi) - \underline{\varphi} [1 - \Phi(\underline{\varphi})] = \int_{\underline{\varphi}}^{\infty} (\varphi - \underline{\varphi}) d\Phi(\varphi) \geq 0$. Hence $\tilde{\varphi}'(\underline{\varphi}) \geq 0$.

A.4 An Example with Labor

In this Appendix we study an example similar to the one in Section 2. Here we use the same setup except that firms produce using technology $y_{jt} = \varphi_j l_{jt}^\alpha$, where l_{jt} denotes the labor input.

For notation ease, here we use π_{jt} to denote *operation profit*, i.e., $\pi_{jt} \equiv \varphi_j l_{jt}^\alpha - w_t l_{jt} - c^f$. Firms take the wage w_t as given and choose the optimal labor input to maximize profit π_{jt} . The optimal labor input, l_{jt}^* , is given by

$$l_{jt}^* = \left(\frac{\alpha \varphi_{jt}}{w_t} \right)^{\frac{1}{1-\alpha}}, \quad (2)$$

The corresponding maximum profit is $\pi_{jt}^* = \frac{1-\alpha}{\alpha} w_t \left(\frac{\alpha \varphi_j}{w_t} \right)^{\frac{1}{1-\alpha}} - c^f$. At a steady state, the value of a potential entrant is given by

$$V_{jt} = \frac{R}{R-p} \pi_j^* + B^e, \quad (3)$$

where π_j^* denotes the maximum profit at the steady state, and B^e is equal to

$$B^e = \begin{cases} B_0, & \text{Bubbly} \\ 0. & \text{Bubble-less} \end{cases} \quad (4)$$

A unit of labor is supplied in a frictionless labor market. At a steady state, labor market clearing implies that

$$\int l_{jt}^* dj = \frac{p}{1-p} \left(\frac{\alpha}{w_t} \right)^{\frac{1}{1-\alpha}} \int_{\underline{\varphi}}^{\infty} \varphi^{\frac{1}{1-\alpha}} d\Phi(\varphi) = 1, \quad (5)$$

where LFS denotes the aggregate labor demand, and RHS denotes the aggregate labor supply. The equilibrium wage is equal to

$$w_t = \alpha \left[\frac{p}{1-p} \int_{\underline{\varphi}}^{\infty} \varphi^{\frac{1}{1-\alpha}} d\Phi(\varphi) \right]^{1-\alpha}, \quad (6)$$

Figure A.1 plots the two conditions that characterize the steady state $\underline{\varphi}$. The upward sloping line represents the relationship between cutoff productivity and profit when labor market clears

$$\begin{aligned} \underline{\pi} &= \frac{1-\alpha}{\alpha} w_t \left(\frac{\alpha \underline{\varphi}}{w_t} \right)^{\frac{1}{1-\alpha}} - c^f \\ &= (1-\alpha) \left[\frac{p}{1-p} \int_{\underline{\varphi}}^{\infty} \varphi^{\frac{1}{1-\alpha}} d\Phi(\varphi) \right]^{-\alpha} \underline{\varphi}^{\frac{1}{1-\alpha}} - c^f, \end{aligned} \quad (7)$$

where both $\left[\frac{p}{1-p} \int_{\underline{\varphi}}^{\infty} \varphi^{\frac{1}{1-\alpha}} d\Phi(\varphi) \right]^{-\alpha}$ and $\underline{\varphi}^{\frac{1}{1-\alpha}}$ are increasing in $\underline{\varphi}$. The horizontal lines correspond to the free entry condition (FE)

$$\frac{R}{R-p} \underline{\pi} + B^e = 0. \quad (8)$$

Bubbles shift the FE horizontal line downward and provide access to the firms with lower productivity. The cutoff productivity $\underline{\varphi}$ declines in the presence of bubbles.

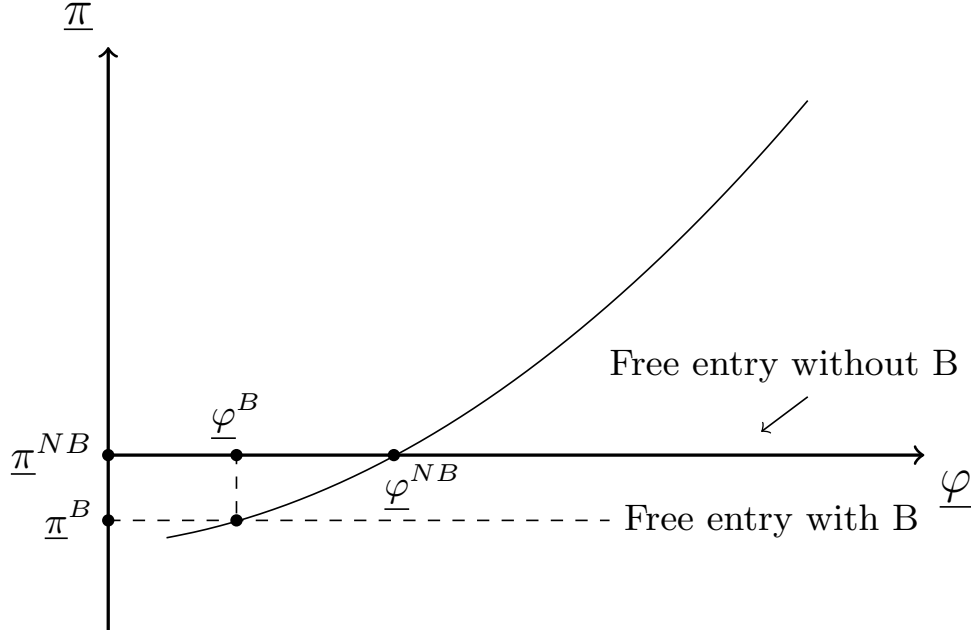


Figure A.1: Cutoff firm at the steady states

We can construct the aggregate production function

$$Y_t \equiv \int y_{jt} dj = \Psi_t L_t^\alpha, \quad (9)$$

where L_t represents the aggregate labor input, which is fixed at 1. Therefore the aggregate productivity is equal to the aggregate output, i.e., $Y_t = \Psi_t$. Firm optimization implies that $w_t L_t = \alpha Y_t$. Hence

$$Y_t = \Psi_t = \left[\frac{p}{1-p} \int_{\underline{\varphi}}^{\infty} \varphi^{\frac{1}{1-\alpha}} d\Phi(\varphi) \right]^{1-\alpha}, \quad (10)$$

Reducing the cutoff productivity $\underline{\varphi}$, bubbles stimulate the aggregate output as well as the aggregate productivity.

As in Section 2, bubbles raise the stock of intangible input φ through encouraging firm entry. The rising stock of intangible input induces rises of the aggregate output, even if the aggregate labor input is fixed.

A.5 The Decomposition of Firms' Values

We have

$$V^c(\lambda, \mu, k) = (1 - \delta)k - k^{(1)} - g(k, k^{(1)}) + \frac{1}{R} \int V(\lambda^{(1)}, \mu^{(1)}, k^{(1)}) dJ(\lambda^{(1)}, \mu^{(1)} | \lambda, \mu), \quad (11)$$

$$V(\lambda, \mu, k) = A\phi k^\alpha - c^f + 1 \{V^c(\lambda, \mu, k) < V^x(k)\} V^x(k) + 1 \{V^c(\lambda, \mu, k) \geq V^x(k)\} V^c(\lambda, \mu, k), \quad (12)$$

where a variable $X^{(i)}$ denotes the variable X after i periods, specifically, $k^{(1)} = k^*(\lambda, \mu, k)$, $k^{(i)} = k^*(\lambda^{(i-1)}, \mu^{(i-1)}, k^{(i-1)})$ for all $i > 1$, k^* denotes the policy function for k .

Moreover, we can expand equation (11) into

$$V^c(\lambda, \mu, k) = (1 - \delta)k - k^{(1)} - g(k, k^{(1)}) + \frac{1}{R} \int [A^{(1)}\phi^{(1)}(k^{(1)})^\alpha - c^f] dJ(\lambda^{(1)}, \mu^{(1)} | \lambda, \mu) \\ + \frac{1}{R} \int_{\Psi^{(1)}} V^x(k^{(1)}) dJ(\lambda^{(1)}, \mu^{(1)} | \lambda, \mu) + \frac{1}{R} \int_{\Phi^{(1)}} V^c(\lambda^{(1)}, \mu^{(1)}, k^{(1)}) dJ(\lambda^{(1)}, \mu^{(1)} | \lambda, \mu). \quad (13)$$

where $\Psi^{(i)} \equiv \{(\lambda^{(i)}, \mu^{(i)}) | V^c(\lambda^{(i)}, \mu^{(i)}, k^{(i)}) < V^x(k^{(i)})\}$,
and $\Phi^{(i)} \equiv \{(\lambda^{(i)}, \mu^{(i)}) | V^c(\lambda^{(i)}, \mu^{(i)}, k^{(i)}) \geq V^x(k^{(i)})\}$.

Define cash inflow $\pi(\lambda, \mu, k)$ as:

$$\pi(\lambda, \mu, k) \equiv (1 - \delta)k - k^{(1)} - g(k, k^{(1)}) + \frac{1}{R} \int [A^{(1)}\phi^{(1)}(k^{(1)})^\alpha - c^f] dJ(\lambda^{(1)}, \mu^{(1)} | \lambda, \mu) \\ + \frac{1}{R} \int_{\Psi^{(1)}} V^x(k^{(1)}) dJ(\lambda^{(1)}, \mu^{(1)} | \lambda, \mu).$$

By repeatedly expanding the last term on the right-hand side of (13), the continuation value $V^c(\lambda, \mu, k)$ can be expressed as

$$V^c(\lambda, \mu, k) = F^c(\lambda, \mu, k) + B, \quad (14)$$

where

$$\begin{aligned}
F^c(\lambda, \mu, k) &\equiv \pi(\lambda, \mu, k) \\
&+ \frac{1}{R} \int_{\Phi^{(1)}} \pi(\lambda^{(1)}, \mu^{(1)}, k^{(1)}) dJ(\lambda^{(1)}, \mu^{(1)} | \lambda, \mu) \\
&+ \frac{1}{R} \frac{1}{R^{(1)}} \int_{\Phi^{(1)}} \int_{\Phi^{(2)}} \pi(\lambda^{(2)}, \mu^{(2)}, k^{(2)}) dJ(\lambda^{(2)}, \mu^{(2)} | \lambda^{(1)}, \mu^{(1)}) dJ(\lambda^{(1)}, \mu^{(1)} | \lambda, \mu) \\
&\vdots \\
&+ \lim_{i \rightarrow \infty} \left(\prod_{j=0}^{i-1} \frac{1}{R^{(j)}} \right) \int_{\Phi^{(1)}} \cdots \int_{\Phi^{(i)}} \pi(\lambda^{(i)}, \mu^{(i)}, k^{(i)}) dJ(\lambda^{(i)}, \mu^{(i)} | \lambda^{(i-1)}, \mu^{(i-1)}) \\
&\cdots dJ(\lambda^{(1)}, \mu^{(1)} | \lambda, \mu),
\end{aligned} \tag{15}$$

$$\begin{aligned}
B &\equiv \lim_{i \rightarrow \infty} \left(\prod_{j=0}^{i-1} \frac{1}{R^{(j)}} \right) \int_{\Phi^{(1)}} \cdots \int_{\Phi^{(i)}} V^c(\lambda^{(i)}, \mu^{(i)}, k^{(i)}) dJ(\lambda^{(i)}, \mu^{(i)} | \lambda^{(i-1)}, \mu^{(i-1)}) \\
&\cdots dJ(\lambda^{(1)}, \mu^{(1)} | \lambda, \mu),
\end{aligned} \tag{16}$$

in which $R^{(0)} = R$.

A.6 The Bubble Component

We need to show:

$$\begin{aligned}
F^c(\lambda, \mu, k) + B &= F^c(\lambda, \mu, k) \\
&+ \lim_{i \rightarrow \infty} \left(\prod_{j=0}^{i-1} \frac{1}{R^{(j)}} \right) \int_{\Phi^{(1)}} \cdots \int_{\Phi^{(i)}} [F^c(\lambda^{(i)}, \mu^{(i)}, k^{(i)}) + B^{(i)}] dJ(\lambda^{(i)}, \mu^{(i)} | \lambda^{(i-1)}, \mu^{(i-1)}) \\
&\cdots dJ(\lambda^{(1)}, \mu^{(1)} | \lambda, \mu).
\end{aligned}$$

Note that we have

$$B = \frac{1}{R} \int_{\Phi'} B' dJ((\lambda', \mu' | \lambda, \mu)), \tag{17}$$

which can be expanded into:

$$\begin{aligned}
B &= \frac{1}{R} \int_{\Phi^{(1)}} B^{(1)} dJ \left(\left(\lambda^{(1)}, \mu^{(1)} | \lambda, \mu \right) \right) \\
&= \frac{1}{R} \frac{1}{R^{(1)}} \int_{\Phi^{(1)}} \int_{\Phi^{(2)}} B^{(2)} dJ \left(\lambda^{(2)}, \mu^{(2)} | \lambda^{(1)}, \mu^{(1)} \right) dJ \left(\lambda^{(1)}, \mu^{(1)} | \lambda, \mu \right) = \dots \\
&= \lim_{i \rightarrow \infty} \left(\prod_{j=0}^{i-1} \frac{1}{R^{(j)}} \right) \int_{\Phi^{(1)}} \dots \int_{\Phi^{(i)}} B^{(i)} dJ \left(\lambda^{(i)}, \mu^{(i)} | \lambda^{(i-1)}, \mu^{(i-1)} \right) \dots dJ \left(\lambda^{(1)}, \mu^{(1)} | \lambda, \mu \right).
\end{aligned} \tag{18}$$

What remains to be proven is:

$$\lim_{i \rightarrow \infty} \left(\prod_{j=0}^{i-1} \frac{1}{R^{(j)}} \right) \int_{\Phi^{(1)}} \dots \int_{\Phi^{(i)}} F^c \left(\lambda^{(i)}, \mu^{(i)}, k^{(i)} \right) dJ \left(\lambda^{(i)}, \mu^{(i)} | \lambda^{(i-1)}, \mu^{(i-1)} \right) \dots dJ \left(\lambda^{(1)}, \mu^{(1)} | \lambda, \mu \right) = 0. \tag{19}$$

According to (15), fundamental component can be written into:

$$\begin{aligned}
F^c(\lambda, \mu, k) &= \pi(\lambda, \mu, k) + \frac{1}{R} \int_{\Phi^{(1)}} \pi \left(\lambda^{(1)}, \mu^{(1)}, k^{(1)} \right) dJ \left(\lambda^{(1)}, \mu^{(1)} | \lambda, \mu \right) \\
&+ \frac{1}{R} \frac{1}{R^{(1)}} \int_{\Phi^{(1)}} \int_{\Phi^{(2)}} \pi \left(\lambda^{(2)}, \mu^{(2)}, k^{(2)} \right) dJ \left(\lambda^{(2)}, \mu^{(2)} | \lambda^{(1)}, \mu^{(1)} \right) dJ \left(\lambda^{(1)}, \mu^{(1)} | \lambda, \mu \right) + \dots \\
&+ \lim_{i \rightarrow \infty} \left(\prod_{j=0}^{i-1} \frac{1}{R^{(j)}} \right) \int_{\Phi^{(1)}} \dots \int_{\Phi^{(i)}} \pi \left(\lambda^{(i)}, \mu^{(i)}, k^{(i)} \right) dJ \left(\lambda^{(i)}, \mu^{(i)} | \lambda^{(i-1)}, \mu^{(i-1)} \right) \dots dJ \left(\lambda^{(1)}, \mu^{(1)} | \lambda, \mu \right) \\
&+ \lim_{i \rightarrow \infty} \left(\prod_{j=0}^{i-1} \frac{1}{R^{(j)}} \right) \int_{\Phi^{(1)}} \dots \int_{\Phi^{(i)}} F^c \left(\lambda^{(i)}, \mu^{(i)}, k^{(i)} \right) dJ \left(\lambda^{(i)}, \mu^{(i)} | \lambda^{(i-1)}, \mu^{(i-1)} \right) \dots dJ \left(\lambda^{(1)}, \mu^{(1)} | \lambda, \mu \right).
\end{aligned}$$

Therefore,

$$\begin{aligned}
F^c(\lambda, \mu, k) &= F^c(\lambda, \mu, k) \\
&+ \lim_{i \rightarrow \infty} \left(\prod_{j=0}^{i-1} \frac{1}{R^{(j)}} \right) \int_{\Phi^{(1)}} \dots \int_{\Phi^{(i)}} F^c \left(\lambda^{(i)}, \mu^{(i)}, k^{(i)} \right) dJ \left(\lambda^{(i)}, \mu^{(i)} | \lambda^{(i-1)}, \mu^{(i-1)} \right) \\
&\dots dJ \left(\lambda^{(1)}, \mu^{(1)} | \lambda, \mu \right).
\end{aligned}$$

It is now straightforward to see that (19) holds. Q.E.D.

A.7 Definition of the Equilibrium

A recursive competitive bubbly equilibrium consists of:¹⁰ i) value functions $V(\lambda_t, \varphi_t, B_t, k_t)$, $V^c(\lambda_t, \varphi_t, B_t, k_t)$, $V^e(\lambda_t, \varphi_t, B_t)$, ii) policy function $k^*(\lambda_t, \varphi_t, B_t, k_t)$, iii) the measure of entrants $\chi_t(\varphi_t, B_t)$, the measure of potential entrants $\omega(\varphi_t, B_t)$, the measure of surviving firms $\eta_t^s(\varphi_t, B_t, k_t)$, the measure of producing firms $\eta_t^p(\varphi_t, B_t, k_t)$, iv) the process of B_t , and v) the investment return R_t , such that for all t ,

1. $V(\lambda_t, \varphi_t, k_t, B_t)$ and $V^c(\lambda_t, \varphi_t, k_t, B_t)$ solve the optimization problem of incumbent firms, which is described by

$$V(\lambda_t, \varphi_t, B_t, k_t) = A_t \varphi_t k_t^\alpha - c^f + \max \{V^c(\lambda_t, \varphi_t, B_t, k_t), V^x(k_t)\},$$

$$V^c(\lambda_t, \varphi_t, B_t, k_t) =$$

$$\max_{k_{t+1}} \left\{ (1 - \delta)k_t - k_{t+1} - g(k_t, k_{t+1}) + \frac{1}{R_t} \int V(\lambda_{t+1}, \varphi_{t+1}, B_{t+1}, k_{t+1}) dJ(\lambda_{t+1}, \varphi_{t+1}, B_{t+1} | \lambda_t, \varphi_t, B_t) \right\},$$

where

$$g(k_t, k_{t+1}) = c_0 \mathbb{1}\{k_{t+1} \neq (1 - \delta)k_t\} k + c_1 \left(\frac{k_{t+1} - (1 - \delta)k_t}{k_t} \right)^2 k_t,$$

$$V^x(k_t) = (1 - \delta)k_t - g(k_t, 0),$$

$k^*(\lambda_t, \varphi_t, B_t, k_t)$ is the associated policy function for optimal k_{t+1} .

2. The value of firm entry is given by

$$V^e(\lambda_t, \varphi_t, B_t) = V^c(\lambda_t, \varphi_t, B_t, 0).$$

3. The process of bubble satisfies

$$B_t = \frac{1}{R_t} \int_{\Phi_{t+1}} B_{t+1} dJ((\lambda_{t+1}, \varphi_{t+1}, B_{t+1} | \lambda_t, \varphi_t, B_t)),$$

where $\Phi_{t+1} \equiv \{(\lambda_{t+1}, \varphi_{t+1}, B_{t+1}) | V^c(\lambda_{t+1}, \varphi_{t+1}, B_{t+1}, k_{t+1}) \geq V^x(k_{t+1})\}$, $k_{t+1} = k^*(\lambda_t, \varphi_t, B_t, k_t)$.

4. The measure of entrants is

$$\chi_t(\varphi_t, B_t) = N_t \int \mathbb{1}\{V^e(\lambda_t, \varphi_t, B_t) \geq 0\} d\omega(\varphi_t, B_t). \quad (20)$$

¹⁰In this Appendix we modify the state space of functions by replacing μ with φ and B . For some functions we keep only the relevant states. We also add time subscripts.

5. The measure of surviving firms follows

$$\begin{aligned}\eta_{t+1}^s(\varphi_{t+1}, B_{t+1}, k_{t+1}) &= \int \mathbb{1}\{V^c(\lambda_{t+1}, \varphi_{t+1}, B_{t+1}, k_{t+1}) \geq V^x(k_{t+1})\} \times \mathbb{1}\{k_{t+1} = k^*(\lambda_t, \varphi_t, B_t, k_t)\} \\ &\quad \times j(\lambda_{t+1}, \varphi_{t+1}, B_{t+1} | \lambda_t, \varphi_t, B_t) d\eta_t^s(\varphi_t, B_t, k_t) \\ &+ \int \mathbb{1}\{V^c(\lambda_{t+1}, \varphi_{t+1}, B_{t+1}, k_{t+1}) \geq V^x(k_{t+1})\} \times \mathbb{1}\{k_{t+1} = k^*(\lambda_t, \varphi_t, B_t, 0)\} \\ &\quad \times j(\lambda_{t+1}, \varphi_{t+1}, B_{t+1} | \lambda_t, \varphi_t, B_t) d\chi_t(\varphi_t, B_t),\end{aligned}\tag{21}$$

where $j(\lambda_{t+1}, \varphi_{t+1}, B_{t+1} | \lambda_t, \varphi_t, B_t)$ denotes the density of transition probability $J(\lambda_{t+1}, \varphi_{t+1}, B_{t+1} | \lambda_t, \varphi_t, B_t)$.

6. The measure of producing firms follows

$$\begin{aligned}\eta_{t+1}^p(\varphi_{t+1}, B_{t+1}, k_{t+1}) &= \int \mathbb{1}\{k_{t+1} = k^*(\lambda_t, \varphi_t, B_t, k_t)\} \times j(\lambda_{t+1}, \varphi_{t+1}, B_{t+1} | \lambda_t, \varphi_t, B_t) d\eta_t^s(\varphi_t, B_t, k_t) \\ &+ \int \mathbb{1}\{k_{t+1} = k^*(\lambda_t, \varphi_t, B_t, 0)\} \times j(\lambda_{t+1}, \varphi_{t+1}, B_{t+1} | \lambda_t, \varphi_t, B_t) d\chi_t(\varphi_t, B_t).\end{aligned}\tag{22}$$

7. The investment return

$$R_t \geq \beta^{-1}.$$

8. The aggregate output is defined by

$$Y_t \equiv \int A_t \varphi_t k_t^\alpha d\eta_t^p(\varphi_t, B_t, k_t).$$

The aggregate equity value is defined by

$$V_t^a \equiv \int V^c(\lambda_t, \varphi_t, B_t, k_t) d\eta_t^s(\varphi_t, B_t, k_t),$$

and the aggregate consumption is given by

$$\begin{aligned}C_t &= Y_t - c^f \int d\eta_t^p(\lambda_t, \varphi_t, B_t, k_t) \\ &\quad - \int [k^*(\lambda_t, \varphi_t, B_t, k_t) - (1 - \delta)k_t - g(k_t, k^*(\lambda_t, \varphi_t, B_t, k_t))] d\eta_t^p(\varphi_t, B_t, k_t) \\ &\quad - \int [k^*(\lambda_t, \varphi_t, B_t, 0) - g(0, k^*(\lambda_t, \varphi_t, B_t, 0))] d\chi_t(\varphi_t, B_t),\end{aligned}$$

which are subject to

$$0 < \frac{V_t^a}{Y_t} < \infty,$$

$$0 \leq \frac{C_t}{Y_t} < \infty.$$

Last but not least, by detrending (20), (21), and (22), we can write down the law of motion for the relative distribution of firms

$$\tilde{\chi}_t(\varphi_t, B_t) = \int \mathbb{1}\{V^e(\lambda_t, \varphi_t, B_t) \geq 0\} d\omega(\varphi_t, B_t). \quad (23)$$

$$\begin{aligned} \tilde{\eta}_{t+1}^s(\varphi_{t+1}, B_{t+1}, k_{t+1}) &= \frac{1}{1+g} \int \mathbb{1}\{V^c(\lambda_{t+1}, \varphi_{t+1}, B_{t+1}, k_{t+1}) \geq V^x(k_{t+1})\} \times \mathbb{1}\{k_{t+1} = k^*(\lambda_t, \varphi_t, B_t, k_t)\} \\ &\quad \times j(\lambda_{t+1}, \varphi_{t+1}, B_{t+1} | \lambda_t, \varphi_t, B_t) d\tilde{\eta}_t^s(\varphi_t, B_t, k_t) \\ &\quad + \int \mathbb{1}\{V^c(\lambda_{t+1}, \varphi_{t+1}, B_{t+1}, k_{t+1}) \geq V^x(k_{t+1})\} \times \mathbb{1}\{k_{t+1} = k^*(\lambda_t, \varphi_t, B_t, 0)\} \\ &\quad \times j(\lambda_{t+1}, \varphi_{t+1}, B_{t+1} | \lambda_t, \varphi_t, B_t) d\tilde{\chi}_t(\varphi_t, B_t), \end{aligned} \quad (24)$$

$$\begin{aligned} \tilde{\eta}_{t+1}^p(\varphi_{t+1}, B_{t+1}, k_{t+1}) &= \int \mathbb{1}\{k_{t+1} = k^*(\lambda_t, \varphi_t, B_t, k_t)\} \times j(\lambda_{t+1}, \varphi_{t+1}, B_{t+1} | \lambda_t, \varphi_t, B_t) d\tilde{\eta}_t^s(\varphi_t, B_t, k_t) \\ &\quad + \int \mathbb{1}\{k_{t+1} = k^*(\lambda_t, \varphi_t, B_t, 0)\} \times j(\lambda_{t+1}, \varphi_{t+1}, B_{t+1} | \lambda_t, \varphi_t, B_t) d\tilde{\chi}_t(\varphi_t, B_t). \end{aligned} \quad (25)$$

where $\tilde{\chi}$, $\tilde{\eta}^s$, and $\tilde{\eta}^p$ respectively denote the relative distribution for entrants, survivors, and producing firms. (23), (24), and (25) are useful for our quantitative analysis.

A.8 The Determination of R

In this appendix we restore the time subscripts of state variables to better demonstrate the proof. Incumbent individuals have the following budget constraint

$$C_{it} + \int_{\Phi_t} \{V^c(\lambda_t, \mu_t, k_t) + [k_{t+1} - (1 - \delta)k_t + g(k_t, k_{t+1})]\} d\theta_{it+1}(\mu_t, k_t) = \int V(\lambda_t, \mu_t, k_t) d\theta_{it}(\mu_t, k_t),$$

where $\theta_{it}(\mu_t, k_t)$ denotes the distribution (CDF) of shares of producing firms. Newly born individuals have the following budget constraint

$$C_{it} + \int_{\Phi_t} \{V^c(\lambda_t, \mu_t, k_t) + [k_{t+1} - (1 - \delta)k_t + g(k_t, k_{t+1})]\} d\theta_{it+1}(\mu_t, k_t) = \int V^e(\lambda_t, \mu_t) d\theta_{it}^e(\mu_t),$$

where $\theta_{it}^e(\lambda_t, \mu_t)$ denotes the distribution (CDF) of entrants. For notation ease, we define

$$V_I^c(\lambda_t, \mu_t, k_t) \equiv V^c(\lambda_t, \mu_t, k_t) + [k_{t+1} - (1 - \delta)k_t + g(k_t, k_{t+1})],$$

where $V_I^c(\lambda_t, \mu_t, k_t)$ denotes the continuation value before investment.

At t , an incumbent household solves the following constrained optimization problem

$$V_{it} = E_{it} \sum_{\tau=t}^{\infty} \beta^{\tau-t} C_{i\tau} - E_{it} \sum_{\tau=t}^{\infty} \Lambda_{i\tau} \left[C_{i\tau} + \int_{\Phi_{\tau}} V_I^c(\lambda_{\tau}, \mu_{\tau}, k_{\tau}) d\theta_{i\tau+1}(\mu_{\tau}, k_{\tau}) - \int V(\lambda_{\tau}, \mu_{\tau}, k_{\tau}) d\theta_{i\tau}(\mu_{\tau}, k_{\tau}) \right],$$

where V_{it} denotes the Lagrangian lifetime utility function for individual i . We can write down a similar Lagrangian function for the utility of any newly born individual i

$$V_{it} = E_{it} \sum_{\tau=t}^{\infty} \beta^{\tau-t} C_{i\tau} - E_{it} \sum_{\tau=t}^{\infty} \Lambda_{i\tau} \left[C_{i\tau} + \int_{\Phi_{\tau}} V_I^c(\lambda_{\tau}, \mu_{\tau}, k_{\tau}) d\theta_{i\tau+1}(\mu_{\tau}, k_{\tau}) - \int V^e(\lambda_{\tau}, \mu_{\tau}) d\theta_{i\tau}^e(\mu_{\tau}) \right],$$

For all t , we have

$$\frac{\partial V_{it}}{\partial \theta_{it+1}} = \Lambda_{it+1} \int V(\lambda_{t+1}, \mu_{t+1}, k_{t+1}) dJ(\lambda_{t+1}, \mu_{t+1} | \lambda_t, \mu_t) - \Lambda_{it} V_I^c(\lambda_t, \mu_t, k_t) = 0. \quad (26)$$

Market for shares can only clear if $\frac{\partial V_{it}}{\partial \theta_{it+1}} = 0$. If $\frac{\partial V_{it}}{\partial \theta_{it+1}} > 0$, $\theta_{it+1} \rightarrow \infty$. If $\frac{\partial V_{it}}{\partial \theta_{it+1}} < 0$, $\theta_{it+1} = -\infty$.¹¹ Equation (26) can be written into

$$V_I^c(\lambda_t, \mu_t, k_t) = \frac{\Lambda_{it+1}}{\Lambda_{it}} \int V(\lambda_{t+1}, \mu_{t+1}, k_{t+1}) dJ(\lambda_{t+1}, \mu_{t+1} | \lambda_t, \mu_t). \quad (27)$$

Equation (27) is indeed the recursive asset pricing Equation (14) in Section 3 of the paper,

¹¹Investors can hold negative amount of shares by short selling.

if firms follow the optimal decision rule of k_{t+1} . The return on equity is given by

$$R_t = \frac{\int V(\lambda_{t+1}, \mu_{t+1}, k_{t+1}) dJ(\lambda_{t+1}, \mu_{t+1} | \lambda_t, \mu_t)}{V_I^c(\lambda_t, \mu_t, k_t)} = \frac{\Lambda_{it}}{\Lambda_{it+1}}. \quad (28)$$

Every firm has to provide the same expected investment return R_t on their shares otherwise the market for shares cannot clear.

For all $\tau \geq t$, we have the following FOC (regardless of whether the individual i is newly born or incumbent)

$$\frac{\partial V_{it}}{\partial C_{i\tau}} = \beta^{\tau-t} - \Lambda_{i\tau}. \quad (29)$$

If $\Lambda_{i\tau} < \beta^{\tau-t}$, $C_{it+1} \rightarrow \infty$. Hence $\Lambda_{i\tau} \geq \beta^{\tau-t}$, and $\frac{\partial V_{it}}{\partial C_{i\tau}} \leq 0$ for all t . Moreover

$$\begin{aligned} \frac{\partial V_{it}}{\partial C_{i\tau}} &= \frac{1}{\beta} (\beta^{\tau+1-t} - \beta \frac{\Lambda_{i\tau}}{\Lambda_{i\tau+1}} \Lambda_{i\tau+1}) \\ &= \frac{1}{\beta} (\beta^{\tau+1-t} - \beta R_\tau \Lambda_{i\tau+1}). \end{aligned} \quad (30)$$

Because $\beta^{\tau+1-t} - \Lambda_{i\tau+1} \leq 0$ for all $\tau \geq t$, if $\beta R_t > 1$, $\frac{\partial V_{it}}{\partial C_{it}} < 0$ and no one consumes at t , i.e., $C_{it} = 0$.¹²

R along a BGP Along a BGP, R is fixed over time and thus we remove the time subscript. Suppose that $R > \beta^{-1}$. As aforementioned, no one ever consumes. For notation ease, we denote individual i 's total wealth with $W_{it} > 0$

$$W_{it} \equiv \begin{cases} \int V(\lambda_t, \mu_t, k_t) d\theta_{it}(\mu_t, k_t), & \text{Incumbents} \\ \int V^e(\lambda_t, \mu_t) d\theta_{it}^e(\mu_t). & \text{Newly born} \end{cases} \quad (31)$$

The investment return is fixed at R . Since there are no consumption and aggregate uncertainty, we have

$$W_{it+1} = RW_{it},$$

¹²Alternatively, one may include the non-negativity constraint $C_{it} \geq 0$ in the Lagrangian function, and impose the FOC $\frac{\partial V_{it}}{\partial C_{i\tau}} = 0$. The analysis is similar and the results are the same.

which implies that

$$\lim_{\tau \rightarrow \infty} \beta^\tau W_{i\tau} = \lim_{\tau \rightarrow \infty} \beta^\tau R^\tau W_{i0} > 0. \quad (32)$$

(32) implies the violation of transversality condition $\lim_{\tau \rightarrow \infty} \beta^\tau W_{i\tau} = 0$, which is necessary for utility optimization. Intuitively, it cannot be optimal if one never consumes.

Demand and Supply of Funds We define net investment as

$$\begin{aligned} \tilde{I}_t \equiv & \int [k^*(\lambda_t, \varphi_t, B_t, k_t) - (1 - \delta)k_t - g(k_t, k^*(\lambda_t, \varphi_t, B_t, k_t))] d\eta_t^p(\varphi_t, B_t, k_t) \\ & + \int [k^*(\lambda_t, \varphi_t, B_t, 0) - g(0, k^*(\lambda_t, \varphi_t, B_t, 0))] d\chi_t(\varphi_t, B_t) \end{aligned} \quad (33)$$

where $k^*(\lambda_t, \varphi_t, B_t, k_t)$ denotes the decision rule for k_{t+1} , $\eta_t^p(\varphi_t, B_t, k_t)$ denotes the distribution of producing firms, and $\chi_t(\varphi_t, B_t)$ denotes the distribution of entrants. $k^*(\lambda_t, \varphi_t, B_t, k_t) = 0$ for exiters. For given $\eta_t^p(\varphi_t, B_t, k_t)$, (33) is graphically depicted by the downward sloping curve in Figure A.2a. Conditional on $\eta_t^p(\varphi_t, B_t, k_t)$, $\tilde{I}_t$ is decreasing in R_t as the number of entrants and $k^*(\lambda_t, \varphi_t, B_t, k_t)$ are decreasing in R_t . (33) can be viewed as the demand for the net investment $\tilde{I}_t$.

Note that aggregate consumption is given by¹³

$$C_t = Y_t - c^f \int d\eta_t^p(\lambda_t, \varphi_t, B_t, k_t) - \tilde{I}_t. \quad (34)$$

Accordingly, we can write down the supply of funds for net investment

$$\tilde{I}_t = \begin{cases} Y_t - c^f \int d\eta_t^p(\lambda_t, \varphi_t, B_t, k_t), & R_t > \beta^{-1} \\ \in [0, Y_t - c^f \int d\eta_t^p(\lambda_t, \varphi_t, B_t, k_t)], & R_t = \beta^{-1} \end{cases} \quad (35)$$

(35) is graphically displayed as the horizontal and vertical line in Figure A.2a, for given $\eta_t^p(\varphi_t, B_t, k_t)$.

The two lines in Figure A.2a together characterize the equilibrium R . We already know that along a BGP, $R = \beta^{-1}$, i.e., the demand curve crosses the horizontal part of the supply curve.

A positive bubble shock at t (an increase in p^b) shifts the demand curve to the right (Figure A.2b). Note that the shock does not shift the supply curve since it does not affect

¹³Equation (34) can also be derived from households' budget constraints.

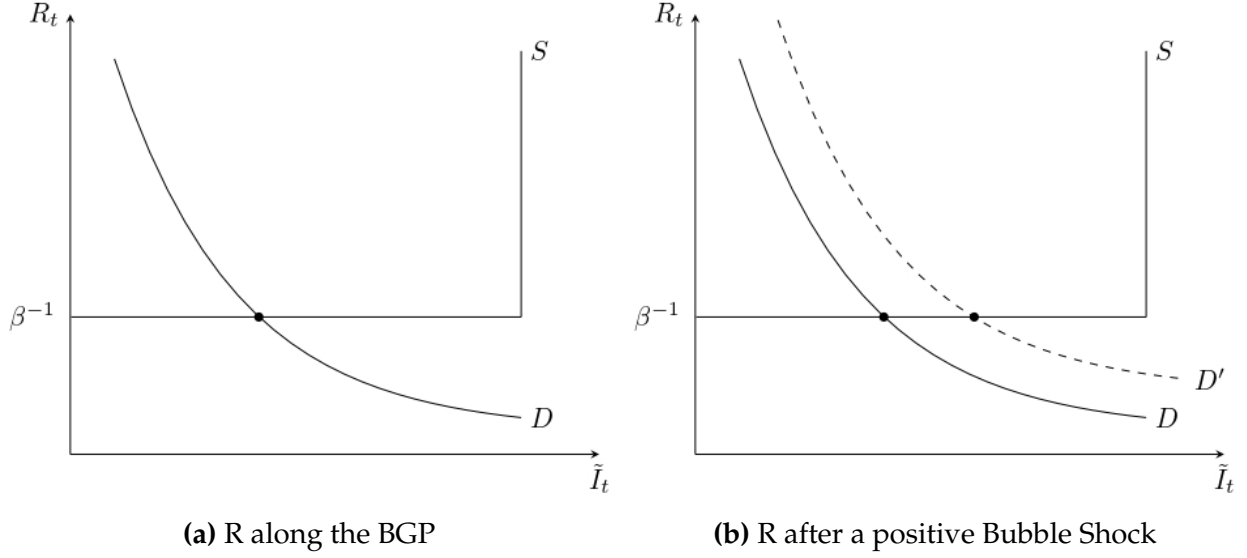


Figure A.2: The determination of R

$\eta_t^p(\varphi_t, B_t, k_t)$ on impact. If the shock is large *enough*, the demand curve crosses the vertical part of the supply curve, $R > \beta^{-1}$, and no one in the economy consumes. We focus on moderate bubble shocks that shift the demand (of funds) curve D without affecting $R = \beta^{-1}$ (Figure A.2b) and depressing the consumption to zero. In our quantitative analysis, we impose $R = \beta^{-1}$ while computing BGP and transition paths; meanwhile the codes verify automatically that the aggregate consumption is positive so that there is no self-contradiction.

A.9 Proof of Proposition 1

In the equity market, the total amount of tradable bubbles B_A is equal to

$$B_A = B_I + B_N,$$

where B_I and B_N denote the amount of bubbles attached to surviving firms and entrants, respectively.

Note that firm-level bubble components follow

$$B' = \begin{cases} 0, & \text{with probability } 1 - p^b \\ \left(\frac{1}{R} \cdot p^b \cdot p^o(\lambda, \mu, k') \right)^{-1} B, & \text{with probability } p^b \end{cases} \quad (36)$$

Since there is no aggregate uncertainty, according to (36), we have

$$\begin{aligned} B'_I &= \int \left[\left(\frac{1}{R} \cdot p^b \cdot p^o(\lambda, \mu, k') \right)^{-1} B \right] \cdot [p^b \cdot p^o(\lambda, \mu, k')] d\eta(\varphi, B, k') \\ &= R \int B \cdot d\eta(\varphi, B, k') = RB_A, \end{aligned} \quad (37)$$

where $\eta(\varphi, B, k')$ represents the distribution of the incumbents and entrants. Therefore, the law of motion for the aggregate bubble is given by

$$B'_A = RB_A + B_0M'. \quad (38)$$

Here, $B_N = B_0M$, where M denotes the number of bubbly entrants. The bubble-output ratio follows

$$\frac{B'_A}{Y'} = R \frac{B_A}{Y} \frac{Y}{Y'} + \frac{B_0M'}{Y'}. \quad (39)$$

On the BGP, both the measure of bubbly entrants M and the aggregate output Y grow at the rate g . Thus, (39) can be rewritten as

$$b' = \frac{R}{1+g}b + b_0, \quad (40)$$

where $b \equiv \frac{B_A}{Y}$ denotes the bubble-output ratio and $b_0 \equiv \frac{B_0M_0}{Y_0}$. The bubble-output ratio follows a stationary process as long as $R < 1 + g$. Indeed, the ratio stays at a constant $\left(1 - \frac{R}{1+g}\right)^{-1} b_0$ on the BGP.

Note that in equilibrium, $R = \beta^{-1}$. Q.E.D.

The proof above does not use the assumption that only p^b of potential entrants are bubbly. Suppose that $p_0^b \in (0, 1]$ of all potential entrants are bubbly. In the law of motion (40), the only term that is dependent on p_0^b is $b_0 \equiv \frac{B_0M_0}{Y_0}$, where $\frac{M_0}{Y_0} \in (0, +\infty)$ for any $p_0^b \in (0, 1]$. The law of motion (40) is stationary if and only if $\beta(1+g) > 1$, regardless of the value of p_0^b .

A.10 Dynamic Inefficiency

Use $\{C_\tau\}_{\tau=0}^\infty$ to denote an arbitrary consumption path for the representative individual of each generation of age τ , along the BGP. We only need to prove that there always exists a Pareto improvement for any arbitrary $\{C_\tau\}_{\tau=0}^\infty$. Normalizing the total population by

setting the size of newly born cohort to one, we get

$$\sum_{\tau=0}^{\infty} \left(\frac{1}{1+g} \right)^{\tau} C_{\tau} = C,$$

where C denotes the aggregate consumption.

Assume that $C_0 > 0$. As will be shown later, this assumption is without loss of generality. Consider a lump-sum transfer scheme that transfers all the consumption of the new cohort to older generations such that

$$\sum_{\tau=1}^{\infty} \left(\frac{1}{1+g} \right)^{\tau} \Delta C_{\tau} = C_0,$$

where $\Delta C_{\tau} > 0$ represents the transfer to the cohort of age τ . The transfer scheme lasts permanently.

It is immediate that all generations of age $\tau > 0$ are better off. As to the newly born, the change in the life-time utility is equal to

$$\Delta U_0 = \sum_{\tau=1}^{\infty} \beta^{\tau} \Delta C_{\tau} - C_0 > \sum_{\tau=1}^{\infty} \left(\frac{1}{1+g} \right)^{\tau} \Delta C_{\tau} - C_0 = 0,$$

provided that $\beta > \frac{1}{1+g}$. Therefore the newly born is also better off.

The reasoning above applies to a consumption path $\{C_{\tau}\}_{\tau=0}^{\infty}$ where $C_0 = 0$ and the youngest generation with positive consumption is of age $\tau = \underline{\tau}$, if we slightly modify the transfer scheme: Now we transfer the consumption of the generation of age $\tau = \underline{\tau}$, i.e., $C_{\underline{\tau}} > 0$, to the generations of age $\tau > \underline{\tau}$

$$\sum_{\tau=\underline{\tau}+1}^{\infty} \left(\frac{1}{1+g} \right)^{\tau} \Delta C_{\tau} = C_{\underline{\tau}}.$$

All the generations of age $\tau \geq \underline{\tau}$ are better off. Meanwhile, younger generations are not worse off. Again, this is a Pareto improvement. Q.E.D.

Such a transfer scheme makes a Pareto improvement to any arbitrary consumption path $\{C_{\tau}\}_{\tau=0}^{\infty}$. It is also very important to note that the transfer scheme does not increase the detrended aggregate consumption C . Hence there is no need of changes in the supply side. In a nutshell, this economy is always dynamic inefficient and there always exists a possible Pareto improvement without increasing the aggregate consumption.

A.11 Proof of Proposition 2

In this appendix we restore the time subscripts of state variables to better demonstrate the proof. The social welfare is the sum of the lifetime utility of all existing households. Incumbent individuals have the following budget constraint

$$C_{it} + \int_{\Phi_t} \{V^c(\lambda_t, \mu_t, k_t) + [k_{t+1} - (1 - \delta)k_t + g(k_t, k_{t+1})]\} d\theta_{it+1}(\mu_t, k_t) = \int V(\lambda_t, \mu_t, k_t) d\theta_{it}(\mu_t, k_t),$$

where $\theta_{it}(\mu_t, k_t)$ denotes the distribution (CDF) of shares of incumbent firms. Newly born individuals have the following budget constraint

$$C_{it} + \int_{\Phi_t} \{V^c(\lambda_t, \mu_t, k_t) + [k_{t+1} - (1 - \delta)k_t + g(k_t, k_{t+1})]\} d\theta_{it+1}(\mu_t, k_t) = \int V^e(\lambda_t, \mu_t) d\theta_{it}^e(\mu_t),$$

where $\theta_{it}^e(\lambda_t, \mu_t)$ denotes the distribution (CDF) of entrants.

For notation ease, define investment portfolio as

$$I_{it} \equiv \int_{\Phi_t} \{V^c(\lambda_t, \mu_t, k_t) + [k_{t+1} - (1 - \delta)k_t + g(k_t, k_{t+1})]\} d\theta_{it+1}(\mu_t, k_t).$$

Since there is no aggregate uncertainty, and the return on the investment portfolio is equal to $R = \frac{1}{\beta}$, the budget constraint at $t + 1$ can be written as

$$C_{it+1} + I_{it+1} = \frac{1}{\beta} I_{it},$$

or

$$I_{it} = \beta(C_{it+1} + I_{it+1}).$$

Hence at t , as for any incumbent individual,

$$\begin{aligned} \int V(\lambda_t, \mu_t, k_t) d\theta_{it}(\mu_t, k_t) &= C_{it} + I_{it} \\ &= C_{it} + \beta(C_{it+1} + I_{it+1}) \\ &= \cdots = U_{it} + \lim_{\tau \rightarrow \infty} \beta^\tau I_{it\tau}. \end{aligned} \tag{41}$$

Imposing transversality condition $\lim_{\tau \rightarrow \infty} \beta^\tau I_{i\tau} = 0$, we get

$$U_{it} = \int V(\lambda_t, \mu_t, k_t) d\theta_{it}(\mu_t, k_t).$$

Similarly, as for newly born individuals,

$$U_{it} = \int V^e(\lambda_t, \mu_t) d\theta_{it}^e(\mu_t).$$

In aggregate

$$U_t = \int V(\lambda_t, \mu_t, k_t) d\eta_t^p(\mu_t, k_t) + \int V^e(\lambda_t, \mu_t) d\chi_t(\mu_t).$$

In our model, the social welfare is equal to the aggregate wealth held by all existing individuals. Q.E.D.

In Figure 3 of the paper we plot the impulse response of welfare following a positive bubble shock. Bubbles have a wealth effect. A positive bubble shock can raise aggregate wealth and therefore lifetime consumption. It might seem obvious that bubbles are distortionary as they act as “subsidies” to firm entry and “taxes” to firm exit. However, this is mistaken. In a bubbly equilibrium, the social benefit of continuation is the value of continuation instead of its fundamental component. Firms make non-distorted decisions when they compares the exit value (the cost of continuation) and continuation value, no matter whether the continuation value is different from its fundamental component.

B Appendix for the Numerical Method

The key to solve the model is to find out fundamental component $F^c(\lambda, \mu, k)$, which follows

$$F^c(\lambda, \mu, k) = (1 - \delta)k - k^{(1)} - g(k, k^{(1)}) + \frac{1}{R} \int F(\lambda^{(1)}, \mu^{(1)}, k^{(1)}) dJ(\lambda^{(1)}, \mu^{(1)} | \lambda, \mu), \quad (42)$$

$$F(\lambda, \mu, k) = A\phi k^\alpha - c^f + 1 \{V^c(\lambda, \mu, k) < V^x(k)\} V^x(k) + 1 \{V^c(\lambda, \mu, k) \geq V^x(k)\} F^c(\lambda, \mu, k). \quad (43)$$

Note that the size of B is predetermined and thus independent to the choice of future capital. (42) can be rewritten into

$$F^c(\lambda, \mu, k) = \max_{k'} \left\{ (1 - \delta)k - k' - g(k, k') + \frac{1}{R} \int F(\lambda', \mu', k') dJ(\lambda', \mu' | \lambda, \mu) \right\}, \quad (44)$$

Equation (44) can also be derived from Equation (14) in the paper if we remove bubbles from both sides of the equation. We can implement value function iteration to solve for $F^c(\lambda, \mu, k)$.

To facilitate our discussion, throughout this appendix we remove the aggregate state λ and replace the idiosyncratic states μ with φ and B . It is worth noting that in our analysis, we take as given that there exist unique $F^c(\varphi, B, k')$, $k^*(\varphi, B, k')$, $p^o(\varphi, B, k')$, and $p^s(\varphi, B, k')$ (defined below) for a given path of B . To check for the uniqueness of the solution, we follow the typical procedure adopted in the literature by verifying that the solution method leads to the same solution given different initial guesses.

Balanced-growth-path Hereafter, we focus on a BGP. Given a set of parameters, the BGP is solved by the following steps.

1. We start by defining grids over idiosyncratic state variables φ , B , and k . As for φ grid, we use 9 points that are exponentially spaced, so that the log productivity points are equally spaced. The process of φ is approximated by Rouwenhorst method.¹⁴ We use 20 equally spaced points for B grid. We construct a denser grid for k since it is chosen by firms: we use 90 points that are exponentially spaced.
2. Make guesses for two functions: 1) fundamental component, $F^c(\varphi, B, k)$; 2) the survival probability in the subsequent period when a firm faces no bubble crash, $p^o(\varphi, B, k')$. $p^o(\varphi, B, k')$ is used to pin down the future bubble, if it does not crash. We interpolate fundamental component $F^c(\varphi', B', k')$ at $B' = \left(\frac{1}{R} \cdot p^b \cdot p^o(\lambda, \mu, k')\right)^{-1} B$, and combine it with the transition probability of φ , to get

$$\begin{aligned} F^e(\varphi, B, k') &\equiv \frac{1}{R} \int F(\varphi', B', k') dJ(\varphi', B' | \varphi, B) \\ &= p^b \frac{1}{R} \int F\left(\varphi', \left(\beta \cdot p^b \cdot p^o(\lambda, \mu, k')\right)^{-1} B, k'\right) \xi(\varphi' | \varphi) \\ &\quad + (1 - p^b) \frac{1}{R} \int F(\varphi', 0, k') \xi(\varphi' | \varphi), \end{aligned}$$

¹⁴See Kopecky and Suen (2010).

where $\xi(\varphi'|\varphi)$ denotes the transition probability of idiosyncratic productivity. $F^e(\varphi, B, k')$ is the expected value $\frac{1}{R} \int F(\varphi', B', k') dJ(\lambda', \mu'|\lambda, \mu)$ in (44), given φ , B , and k' .

3. Interpolate $F^e(\varphi, B, k')$ at $k' = (1 - \delta)k$, and calculate the continuation value if a firm does not adjust its capital stock.

4. Calculate the continuation value if a firm adjusts its capital stock, and find out the associated optimal k' .

5. Compare values from 3 and those from 4, update the guess of continuation value, and characterize the policy function for k' .

6. Provided the new guess of $F^c(\varphi, B, k)$ and the transition probability of φ , we can compute survival probability

$$p^s(\varphi, B', k') \equiv \int_{\Phi^s} d\xi(\varphi'|\varphi),$$

where $\Phi^s \equiv \{\varphi' | V^c(\varphi', B', k') \geq V^x(k')\}$. Interpolate $p^s(\varphi, B', k')$ at $B' = \left(\frac{1}{R} \cdot p^b \cdot p^o(\varphi, B, k')\right)^{-1} B$ to update the guess of $p^o(\varphi, B, k')$.

7. Repeat Step 3-6 until the guesses converge.

8. Substitute k' in $p^o(\varphi, B, k')$ with the policy function, and denote the new function with $\tilde{p}^o(\varphi, B, k)$. Now given the transition matrix of φ , $\tilde{p}^o(\varphi, B, k)$, and the policy function for k' , we can construct the transition matrix, Γ , of idiosyncratic states φ , B , and k . Combining the transition matrix with the detrended vector of entrants, $\tilde{\xi}$, we can characterize the stationary distribution vector of φ , B , and k . Iterate the following equation¹⁵

$$\tilde{\eta}_{t+1} = \frac{1}{1+g} \Gamma \times \tilde{\eta}_t + \tilde{\xi}$$

until $\tilde{\eta}_t$ converges. The fixed point of $\tilde{\eta}_t$ is a detrended vector of the stationary distribution on the BGP.

Transition Dynamics Transition dynamics are perfect foresight equilibrium paths following unexpected shocks. Prior to $t = 0$, the economy is at the BGP. Shocks are modeled as innovations to p^b , β , or A at $t = 0$, and the innovations gradually decay to zero. We assume that the economy returns to the BGP at $t = 100$.

Since p^b and A follow deterministic processes after the shocks are realized, and $F_t^c(\varphi_t, B_t, k_t)$ at $t = 100$ is equal to the fundamental component $F^c(\varphi, B, k)$ along the BGP, we can use backward induction to pin down fundamental component $F_t^c(\varphi_t, B_t, k_t)$, probability

¹⁵This corresponds to Equation (24) in A.7.

$p_t^o(\varphi_t, B_t, k_{t+1})$, and policy function $k_{t+1}^*(\varphi_t, B_t, k_t)$ back from $t = 99$ to $t = 0$. Starting from $t = 99$

1. We firstly make a guess regarding $p_t^o(\varphi_t, B_t, k_{t+1})$, which is used to predict the associated bubble size at $t + 1$. Provided $F_{t+1}^c(\varphi_{t+1}, B_{t+1}, k_{t+1})$ and the transition probability of φ , we can compute probability

$$p_t^s(\varphi_t, B_{t+1}, k_{t+1}) \equiv \int_{\Phi_{t+1}^s} d\zeta(\varphi_{t+1}|\varphi_t),$$

where $\Phi_{t+1}^s \equiv \{\varphi_{t+1} | V_{t+1}^c(\varphi_{t+1}, B_{t+1}, k_{t+1}) \geq V^x(k_{t+1})\}$. Interpolate $p_t^s(\varphi_t, B_{t+1}, k_{t+1})$ at $B_{t+1} = \left(\frac{1}{R_t} \cdot p^b \cdot p_t^o(\varphi_t, B_t, k_{t+1})\right)^{-1} B_t$ to update the guess of $p_t^o(\varphi_t, B_t, k_{t+1})$. Repeat this process until the guess converges.

2. Use backward induction to compute $F_t^c(\varphi_t, B_t, k_t)$ and $k_{t+1}^*(\varphi_t, B_t, k_t)$.

3. Iterate (1)-(2) back from $t = 99$ to $t = 0$.

Finally, using fundamental component $F_t^c(\varphi_t, B_t, k_t)$, probability $p_t^o(\varphi_t, B_t, k_{t+1})$, and policy function $k_{t+1}^*(\varphi_t, B_t, k_t)$, we compute the transition matrix Γ_t and the vector of entrants $\tilde{\zeta}_t$ from $t = 0$ to $t = 99$. We can characterize the distribution of idiosyncratic states along the transition dynamics following

$$\tilde{\eta}_{t+1} = \frac{1}{1+g} \Gamma_t \times \tilde{\eta}_t + \tilde{\zeta}_{t+1},$$

where $\tilde{\eta}_0$ is the stationary distribution on the BGP.

C Appendix for Quantitative Results

C.1 The Effects of Bubbles along Intensive Margin

Figure C.1 plots the decision rule of k_{jt+1} as functions of φ_{jt} and k_{jt} , given different B_{jt} . We denote the functions by $k_{B_{jt}=X}^*(\varphi_{jt}, k_{jt})$. B_{max} represents the largest grid of B_{jt} , which we set to be equal to the largest grid of k_{jt} . It can be seen that the size of B_{jt} does not affect the optimal k_{jt+1} .

Note that in the computation we use grid-search to determine the optimal k_{jt+1} . We may not conclude from Figure C.1 that bubbles have *zero* effects on investment along the intensive margin, because we use discrete grids. Yet we can infer from the quantitative

analysis that bubbles do not have substantial effects on investments along the intensive margin.

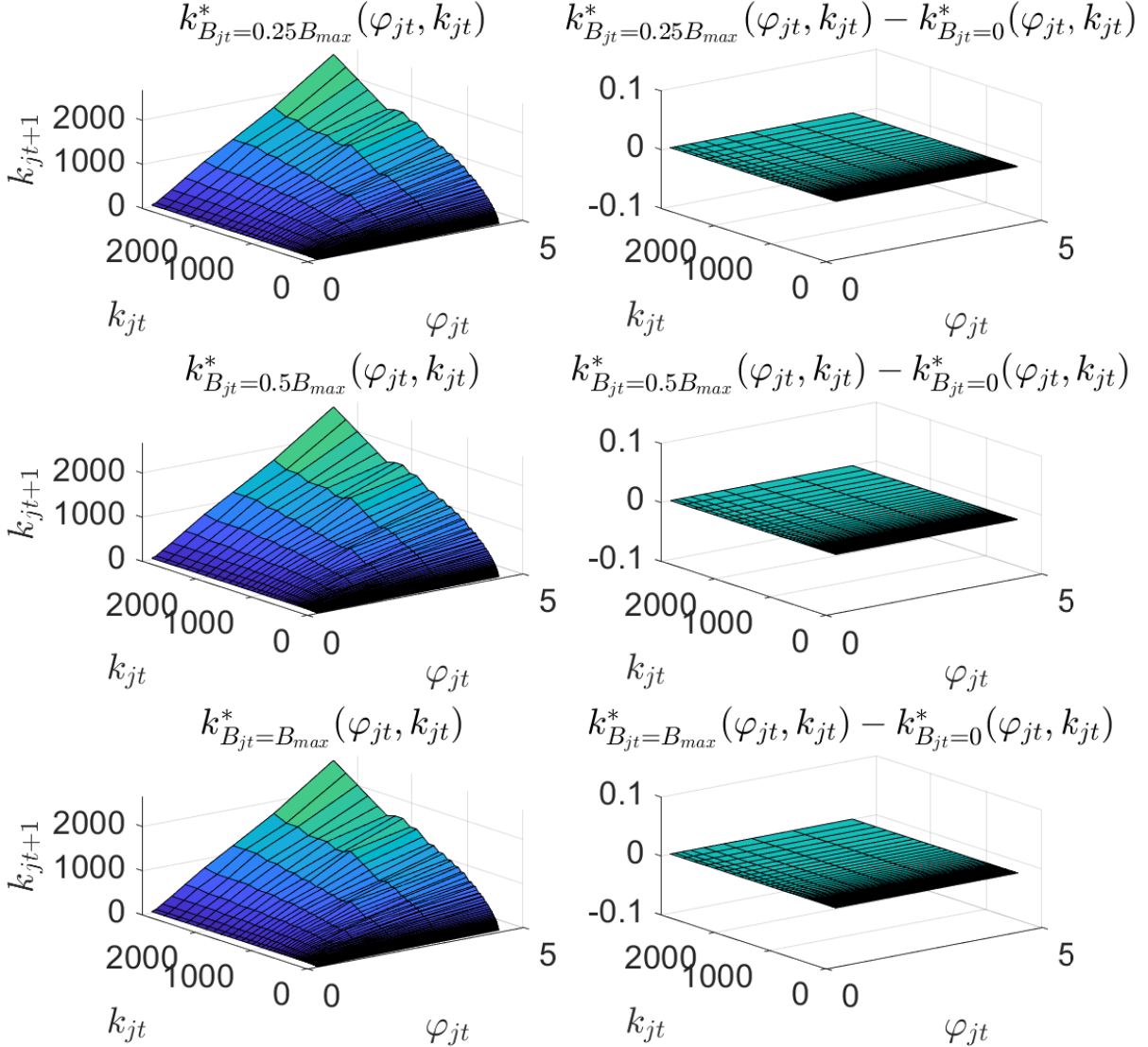


Figure C.1: Policy functions for k_{t+1} given different values of B_t

C.2 Decomposition of the Impulse Responses

The bubble shock we study in Section 4.2 raises the number of bubbly firms. In particular, the group of firms are joined by the bubbly firms that would have exited and the bubbly

firms that would have not entered. These firms, as we can infer from Section 4.1, are on average smaller and less productive relative to their bubbleless counterparts. Because of mean-reverting of idiosyncratic productivity, these bubbly firms tend to outgrow their bubbleless counterparts in subsequent periods and contribute to slow build-up of aggregate production, which exhibits a slow-moving hump shape.

Below we present a decomposition exercise to study how firm entry and exit contribute to the fluctuations of the aggregate output. The decomposition sheds light on the slow-moving and hump-shaped effects of bubble shocks on the aggregate production.

Decomposition For any $t > 0$, changes in the aggregate output is equal to

$$Y_t - Y_0 = - \int_{\Phi_t^X} y_{j0} dj + \int_{\Phi_t^S} (y_{jt} - y_{j0}) dj + \int_{\Phi_t^N} y_{jt} dj, \quad (45)$$

where Φ_t^X denotes the set of firms that produce at 0 and exit before t , Φ_t^S denotes the set of firms that survive from 0 to t , Φ_t^N denotes the set of firms that enter the market after 0 and produce at t . (45) can be rewritten into

$$Y_t - Y_0 = -Y_t^X + Y_t^S + Y_t^N, \quad (46)$$

where $Y_t^X \equiv \int_{\Phi_t^X} y_{j0} dj$, $Y_t^S \equiv \int_{\Phi_t^S} (y_{jt} - y_{j0}) dj$, and $Y_t^N \equiv \int_{\Phi_t^N} y_{jt} dj$. Moreover, we use the sup-script B to denote the variables along the transition dynamics following the bubble shock in Section 4.2, e.g., $Y_t^{N,B}$ denotes Y_t^N along the transition path following the bubble shock. We use the sup-script BGP to denote the variables along the BGP, e.g., $Y_t^{N,BGP}$ denotes Y_t^N along the BGP.

Equation (45) implies that the growth of aggregate output depends on: 1) the *cumulative* output loss from firm exit, which is measured by Y_t^X , 2) the output growth of survivors, which is measured by Y_t^S , and 3) the output contribution of new firms, which is measured by Y_t^N . Our target of decomposition, $Y_t^B - Y_t^{BGP}$, is equal to

$$Y_t^B - Y_t^{BGP} = - \left(Y_t^{X,B} - Y_t^{X,BGP} \right) + \left(Y_t^{S,B} - Y_t^{S,BGP} \right) + \left(Y_t^{N,B} - Y_t^{N,BGP} \right). \quad (47)$$

Figure C.2 displays the impulse responses of Y_t , Y_t^X , Y_t^S , and Y_t^N , measured by their deviations from their BGPs, following the bubble shock. Note that the first panel is different from the impulse response of output in Figure 3 in the paper, which plots the log of output instead of the levels.

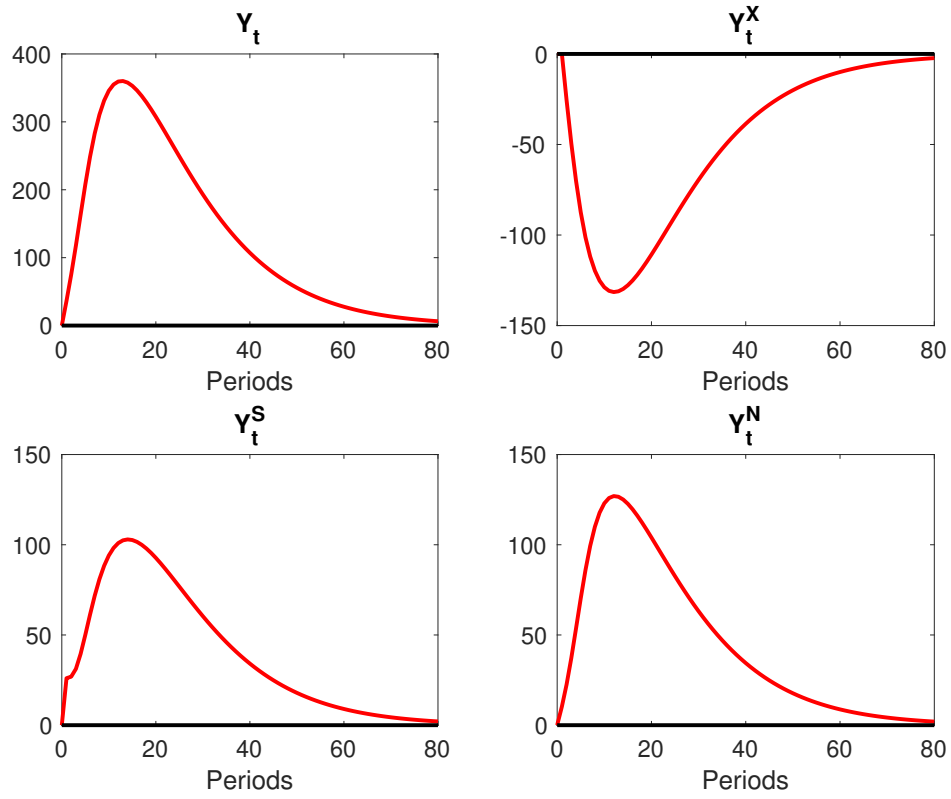


Figure C.2: Decomposition of the Impulse Response Function of Output

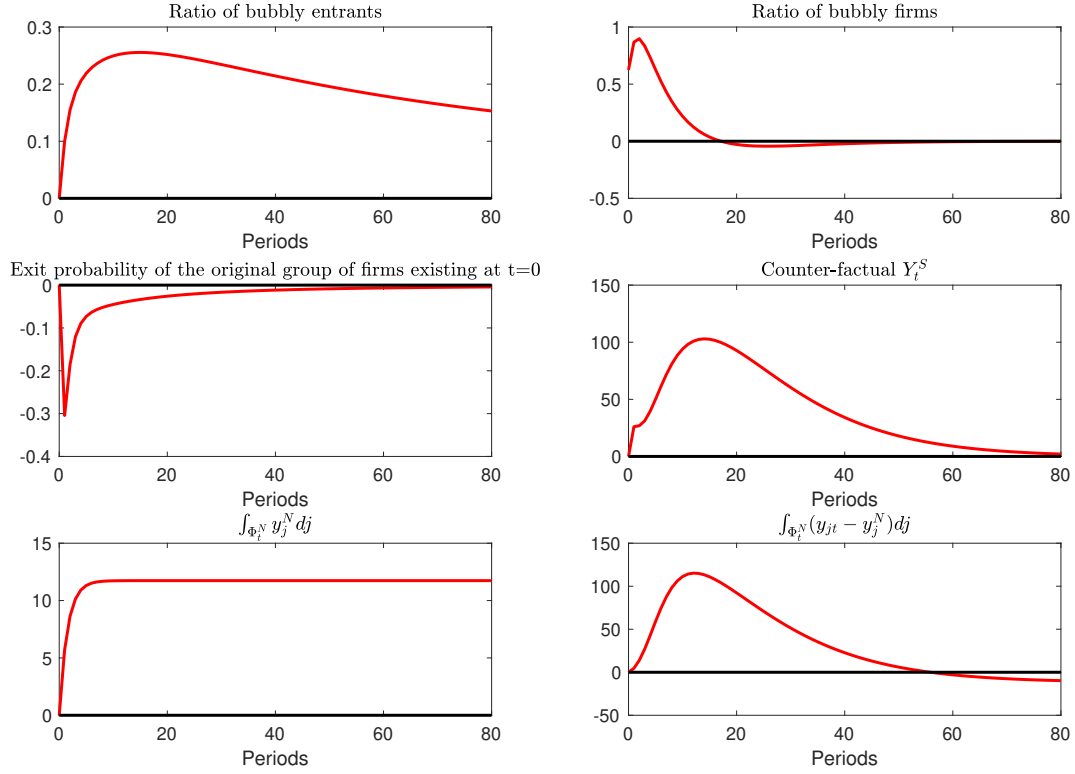


Figure C.3: This figure plots the IRFs that are related to Figure C.2. All the IRFs are measured as the deviation from the BGP and normalized as done in Section 4.2.

Firm Exit The bubble shock raises the fraction of bubbly firms among the firms that produce at 0. This fraction decays slowly, as shown by the first row of Figure C.3. Note that bubbly firms are less likely to exit. The declining firm exit gives rise to: 1) less output loss from firm exit, and 2) changing composition of the survivors.

The second and third panel of Figure C.2 capture the two transmission mechanisms of firm exit. Firstly, Y_t^X measures the *cumulative* direct output loss from the exit of the firms that produce at 0. Following the bubble shock, the decline in firm exit results in less output loss. Since the original group of firms (which produce at t) has less exit along the transition path. (See Figure C.3) The effect adds up over time and thus exhibit a hump shape, before it slowly decays to zero: In the long run, all the firms that produce at 0 exit, thus Y_t^X converges to its BGP asymptotically.

Secondly, the impulse responses of Y_t^S captures the indirect effect of the bubble shock through reducing firm exit. Along the transition path, the survivors include the firms that would survive regardless of the shock, and the bubbly firms that would have exited without the shock. As aforementioned, the bubbly firms that would have exited are on

average less productive. Due to mean-reverting property of idiosyncratic productivity, the less-productive firms tend to grow, which contribute positively to the total output of survivors. Such growth induces Y_t^S to rise over time until the less-productive firms become sufficiently productive. Since the bubble shock generates a persistent rise in the fraction of bubbly firms, it adds extra momentum to the slow buildup of the hump shape. In the long run, as all the firms that produce at 0 exit, Φ_t^S diminishes to empty, and Y_t^S converges to the BGP (zero) asymptotically.

One may suspect that the decision rule of investment may change over the transition path, and such change may contribute to the impulse responses of Y_t^S . Figure C.3 plots the counter-factual impulse response of Y_t^S following the bubble shock, if firms follow the same decision rule of investment as they do along the BGP. The impulse response is almost identical to the one in Figure C.2. The result suggests that the impulse responses of Y_t^S in Figure C.2 is almost completely driven by the changes in the composition of firms instead of the changes in the decision rule of investment.

Firm Entry The last panel in Figure C.2 captures the effects of firm entry. Similar to firm exit, firm entry contributes to the propagation through two different channels. On the one hand, the increasing firm entry directly increase the number of firms and stimulate the aggregate output. The effect slowly accumulates over the transition path. On the other hand, compared with the entrants along the BGP, the entrants following the bubble shock are less productive on average. Because of mean-reverting of idiosyncratic productivity, the entrants that would have not entered without the shock tend to grow afterwards. This channel gives rise to the slow-moving and hump-shaped increase in the aggregate output before it slowly decays due to the gradual exit of the firms that would have not entered. Figure C.3 demonstrate the two different channels.

The last panel in the left column of C.3 plots the cumulative direct effect of firm entry, $\int_{\Phi_t^N} y_j^N dj$, where y_j^N represents the output of firm j while entering the market. There are more entrants along the transition path relative to the BGP. Hence the effects accumulate over time. The last panel in the right column plots $\int_{\Phi_t^N} (y_{jt} - y_j^N) dj$. In the long run the increased entrants along the transition path all exit: As to these firms, $y_{jt} - y_j^N \rightarrow -y_j^N$. Hence the effect converges to a negative amount in the long run.

D Additional Results and Discussions: Empirical Part

D.1 Empirical Method: Discussions

Our Measure of Bubble v.s Hanson (2022)’s Measure The innovation of the current paper is to identify bubble shocks. The validity of the identified shocks depends crucially on the constructed price-fundamental differential component. This paper follows the approach proposed by Galí and Gambetti (2015) closely. In a more recent paper, Hanson (2022) extended Galí and Gambetti (2015)’s model to allow nonlinear, non-Gaussian dynamics. Hanson (2022) estimated the latent bubble components using the Bayesian method with a particle filter. Figure D.1 plots our measure of the price-fundamental differential (solid red line) together with Hanson (2022)’s estimates (blue dash line).¹⁶ Both series are demeaned. The two series are very similar (a correlation of 0.91), despite the complexity introduced in Hanson (2022). The estimated price-fundamental differential component captures the well-known dot-com (around 2000) and housing bubble episodes (around 2008).¹⁷ The non-fundamental component showed a pronounced boom during the build-up phases and the collapse of these bubbles.

¹⁶The authors thank Andrew Hanson for providing his estimates.

¹⁷See, e.g., Brunnermeier *et al.* (2020) for the identification of these bubble dates.

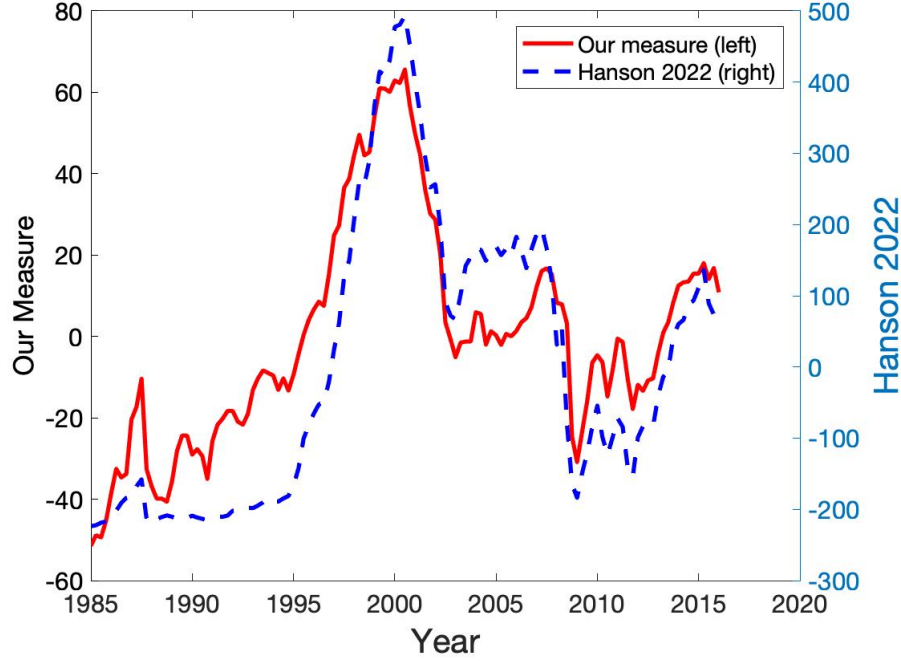


Figure D.1: This figure plots our measure of the price-fundamental differential (solid red line) together with [Hanson \(2022\)](#)'s estimates (blue dash line). Both series are demeaned. The correlation between the two series is 0.91.

The Constructed Price-fundamental Differential The constructed measure of $p_t - f_t$ does not necessarily coincide with the true price-fundamental differential for two reasons. First, the VAR's forecasts of d_{t+h} and r_{t+h} at time t might be different from the rational expectations of the corresponding variables. Second, in the empirical application, we use the real rate constructed from Moody's Baa Corporate Bond Yield to approximate the discount factor.¹⁸ As will be discussed later, the medium-run restriction searches for the shock that maximizes the forecast error variance decomposition of our proxy of the non-fundamental component, controlling for fundamental shocks. Despite the proxy of the price-fundamental differential being merely a proxy, the identification strategy remains valid as long as the uncontrolled fundamental shocks are not as important as bubble shocks in explaining our proxy of the non-fundamental component. Moreover, in Section [D.2](#), we conduct robustness checks controlling for risk premium shocks, monetary policy shocks, and credit supply shocks; the results are similar to our baseline findings.

There are two points to be clarified. First, our identification strategy targets the *proxy*

¹⁸[Galí and Gambetti \(2015\)](#) use risk free rate as the discount factor. Our empirical findings are robust to this alternative choice.

of the non-fundamental component instead of the true non-fundamental component. The noise introduced in the construction of the price-fundamental component does not detract the identification of bubble shocks if the latter explain most of the fluctuations in the constructed non-fundamental component. Section D.2 conducts robustness checks by including additional variables—measures of risk premium, credit supply, and monetary policy—into the baseline VAR. Estimation results confirm that the baseline findings are robust to alternative proxies of the non-fundamental component arising from different VAR specifications.

Second, in the baseline specification, we do allow other shocks, such as risk premium shocks, to have significant impacts on *asset prices*. The validity of the identification requires the response of the proxy of the *fundamental component* to be similar to the response of *asset prices* so that the overall response of the constructed price-fundamental differential is small. Nevertheless, there might exist fundamental shocks that do not conform to this requirement. To address this issue, we control for productivity shocks (unanticipated and anticipated) in the baseline, and shocks to interest rate, risk premium, and credit supply in robustness checks. Overall, the empirical findings are similar both qualitatively and quantitatively across specifications. More importantly, the empirical effects uncovered are consistent with the impulse responses implied by the model.

D.2 Robustness Checks

This section demonstrates that the baseline empirical findings are robust to alternative specifications that control for risk premium shocks, monetary policy shocks, and credit supply shocks, separately.

Risk Premium Shocks The main concern about the identification strategy in the baseline specification is that there might exist other fundamental shocks that contribute significantly to the fluctuations in the constructed price-fundamental differential component. One important candidate of the confounding shocks is the risk premium shock.

One might question the validity of the identification strategy basing on the argument that the risk premium shock is one of the main drivers of *asset price* fluctuations. This criticism is not entirely accurate. For the risk premium shock to be troublesome, it needs to have significant contribution to the variations in the *proxy of the price-fundamental differential component*. In the baseline specification, we do allow for the possibility that risk premium shocks *can* have significant impacts on asset prices. The implicit assumption

made in the baseline analysis is that the response of the proxy of the fundamental component to risk premium shocks is similar to the asset prices' response so that the overall response of the constructed price-fundamental differential is small. Nevertheless, this might still be a strong assumption.

To address this concern, we conduct a robustness check controlling for risk premium shocks. Specifically, we control for shocks that have impact effects on the Volatility Index (VIX). Carr and Wu (2009) and Bekaert *et al.* (2013) document that the VIX contains a risk premium component that reflects risk aversion in financial markets. Therefore, controlling for the VIX mitigates the confounding problem with risk premium shocks described above. If omitting risk premium were indeed a problem, we would expect to see big changes in the estimated IRFs; however, Figure D.2 shows that this is not the case. By controlling for risk premium shocks, the estimated IRFs, particularly the responses of the key variables that this article emphasizes—real GDP and the total number of firms—are similar to those obtained in the baseline.

Monetary Policy Shocks Omitting monetary policy shocks could also lead to the confounding problem. To address this, we construct "bubble shocks" controlling for shocks to the short-term interest rate. Figure D.3 presents the estimation results: the IRFs of the key variables are similar to those obtained in the baseline model.

Other Financial Shocks: the Credit Supply Shocks Shocks to the credit market—the excess bond premium measure (credit supply shocks) proposed by Gilchrist and Zakrajsek (2012)—can have large effects on asset prices, and possibly, contribute to the fluctuations in the constructed price-fundamental differential component significantly. We construct the "bubble shocks" by controlling for shocks that have impact effects on the excess bond premium. Figure D.4 shows that the main message of the paper remains unchanged. Finally, Figures D.5 and D.6 show that the empirical findings are robust to alternative calibrations of $\frac{\Gamma}{R}$.

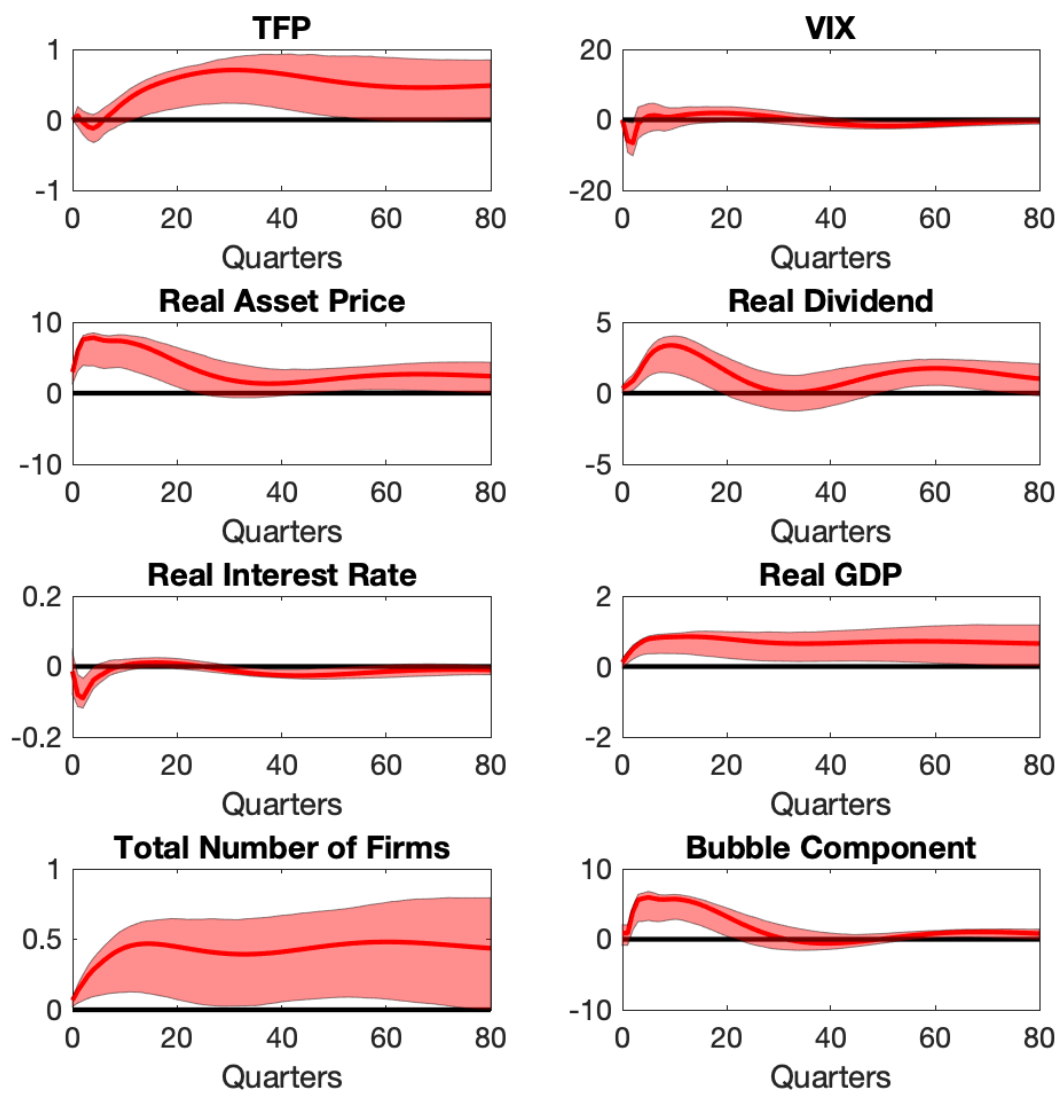


Figure D.2: Robustness check I: controlling for risk premium shocks. This figure plots the impulse response functions to a one standard deviation "bubble shock" estimated based on the a SVAR that controls for TFP shocks and risk premium shocks. The shaded areas indicate the 95% confidence bands.

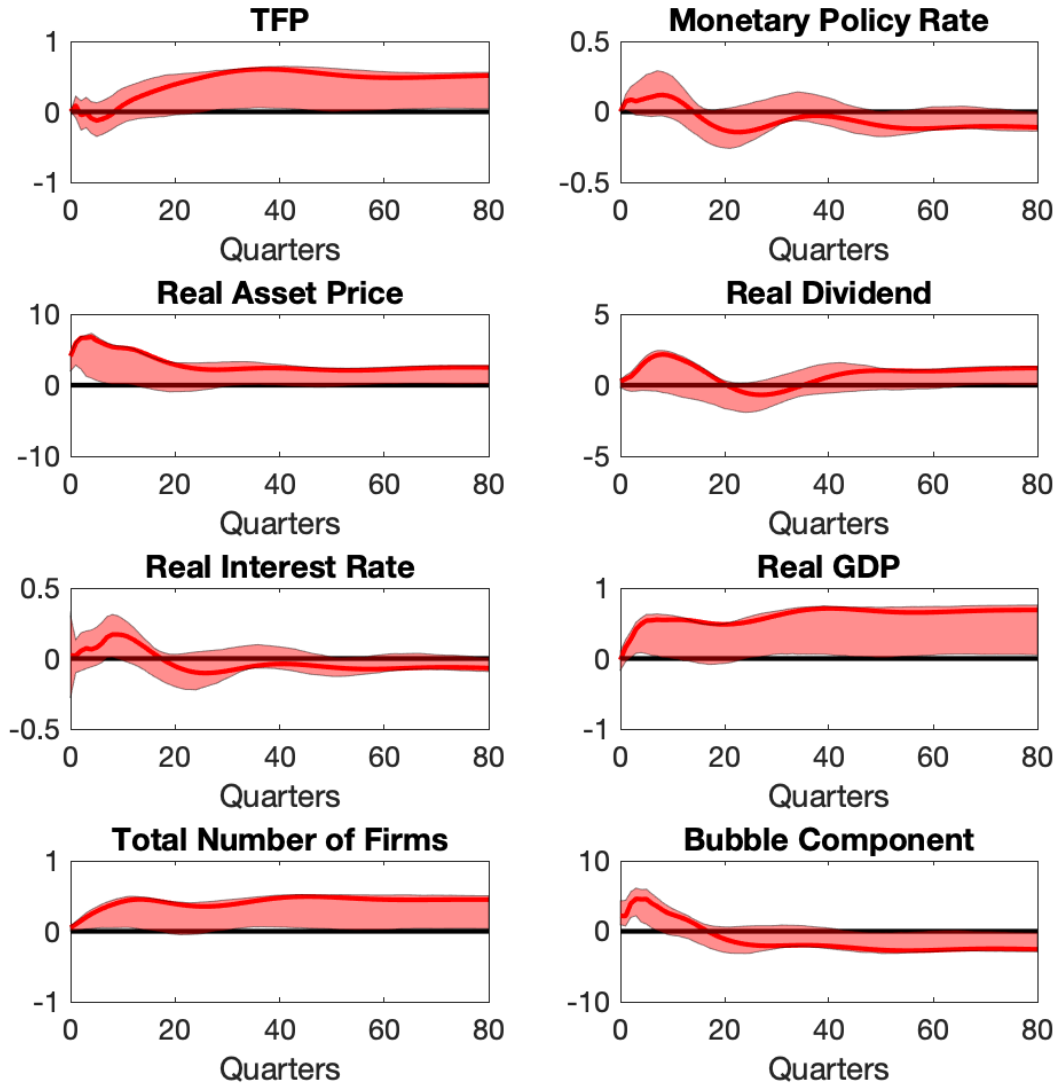


Figure D.3: Robustness check II: controlling for shocks to the short-term interest rate. This figure plots the impulse response functions to a one standard deviation "bubble shock" estimated based on the a SVAR that controls for TFP shocks and shocks to the short-term interest rate. The shaded areas indicate the 95% confidence bands.

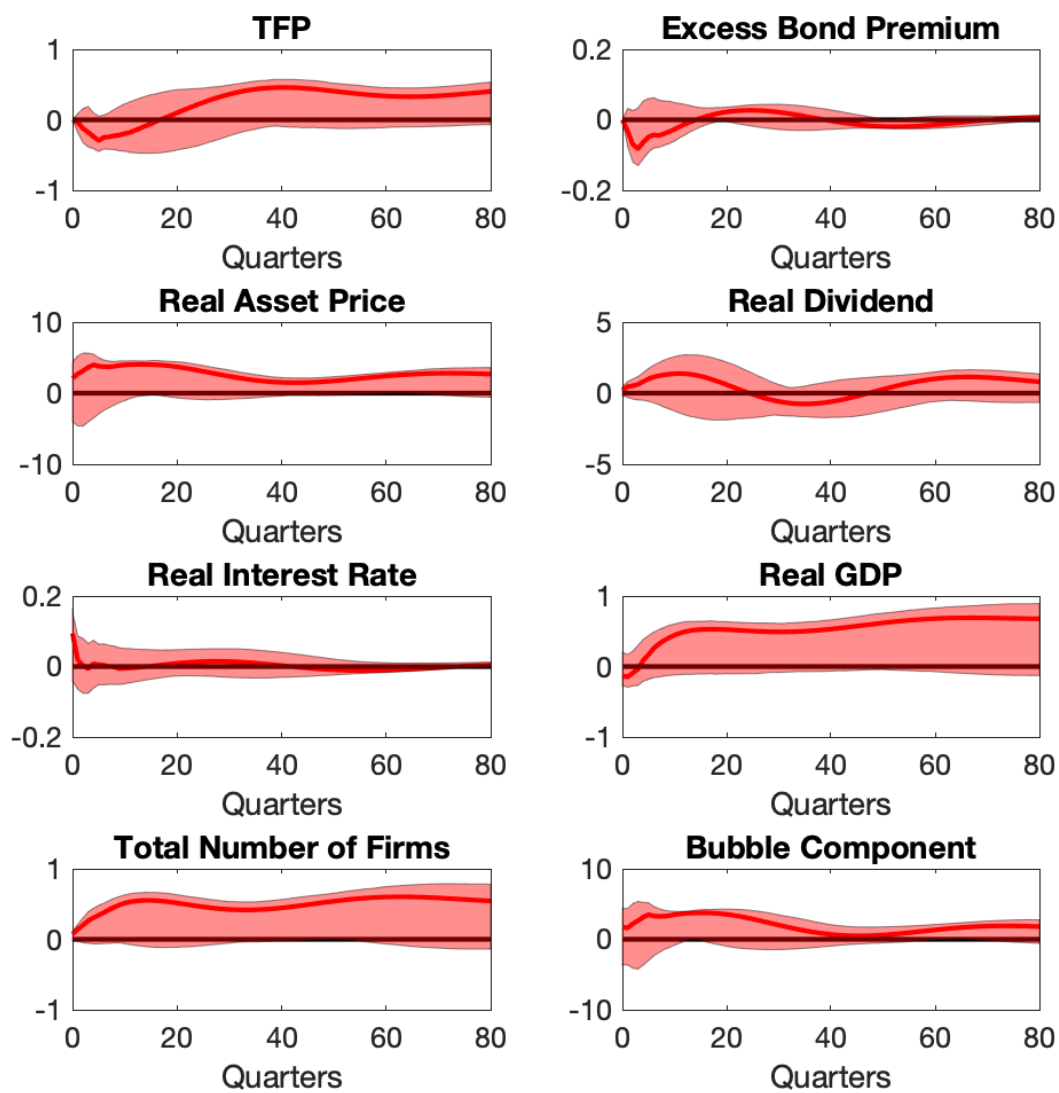


Figure D.4: Robustness check III: controlling for credit supply shocks. This figure plots the impulse response functions to a one standard deviation "bubble shock" estimated based on the a SVAR that controls for TFP shocks and credit supply shocks. The shaded areas indicate the 95% confidence bands.



Figure D.5: Robustness Check IV: using alternative calibration of $\frac{\Gamma}{R} = 0.99$. This figure plots the impulse response functions to a one standard deviation "bubble shock" estimated based on the SVAR. The shaded areas indicate the 95% confidence bands.



Figure D.6: Robustness Check V: using alternative calibration of $\frac{\Gamma}{R} = 0.75$. This figure plots the impulse response functions to a one standard deviation "bubble shock" estimated based on the SVAR. The shaded areas indicate the 95% confidence bands.

D.3 Additional Empirical Results and Interpretations

Effects of bubbles along the Intensive and Extensive Margins Models in the existing literature focus on the intensive margin. They predict that asset bubbles raise the aggregate output through the relaxation of financial frictions without affecting the number of firms in the economy.¹⁹ According to these models, the output per firm increases in response to positive bubble shocks.

It has been highlighted in the current paper that bubble shocks' effects on the total number of firms uncovered in the data support the relevance of the extensive margin. However, it can be inferred from Figure 3 in the paper that output per firm increases in response to a positive "bubble shock": output increases by about 1% whereas the total number of firms increases by 0.5%. This is confirmed in Figure D.7, which reports the responses of output per firm. This evidence suggests that the transmission of bubbles along the intensive margin (models with financial frictions) is important too.

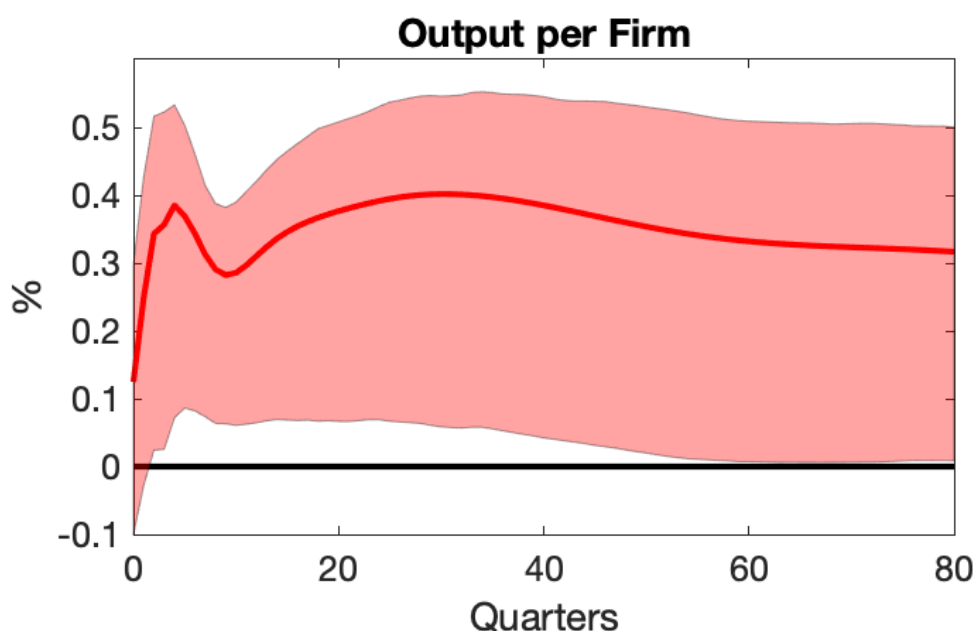


Figure D.7: This figure plots the impulse response function of output per firm to a one standard deviation "bubble shock" estimated based on the baseline SVAR. The shaded areas indicate the 95% confidence bands.

Overall, the IRFs of the total number of firms and the output per firm to "bubble shocks" constructed using aggregate data support the existence of both the extensive and the intensive margin.

¹⁹See, for example, Farhi and Tirole (2012), Martin and Ventura (2012), Miao and Wang (2018), and Ikeda and Phan (2019).

The following empirical exercise illustrates one event study—the dot-com boom that went burst—during which changes in output per firm are consistent with the extensive margin of asset bubbles. Figure D.8 plots the output per firm for the information sector (NAICS 51).²⁰ Interestingly, the output per firm decreases during the build-up phase of the dot-com boom. Moreover, output per firm started to increase as soon as the dot-com bubble burst.

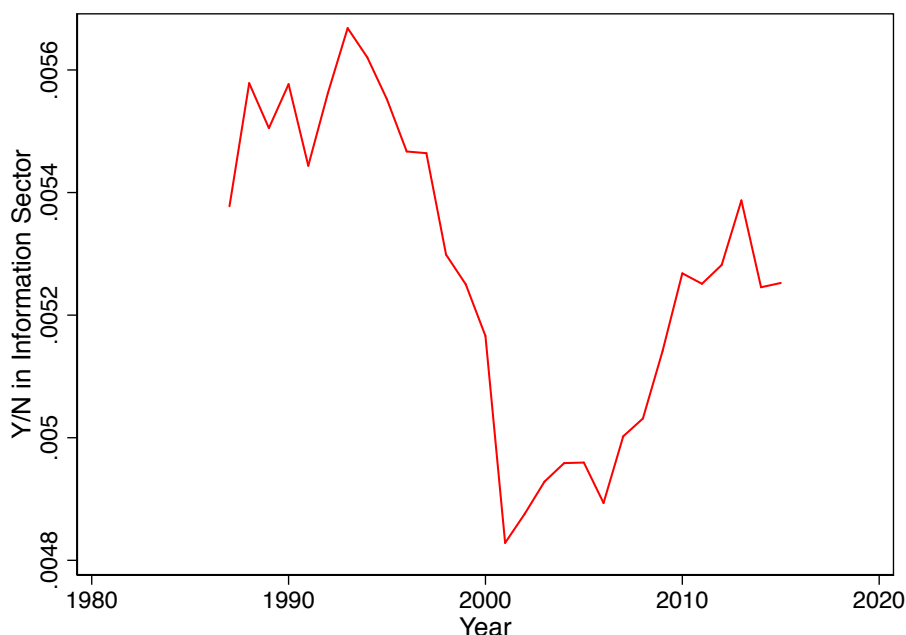


Figure D.8: This figure plots the quantity per number of establishments (Y^{IT}/N^{IT}) in the information sector (NAICS 51). We use the total consumption expenditure of goods produced by the information sector as a proxy for quantity.

Alternative Interpretation of Empirical Findings The empirical findings support the transmission channel of rational bubbles discussed in this study; however, our model may not be the only explanation of the data. Section D.2 presents evidence that three well-known financial shocks—risk premium shocks, credit supply shocks, and shocks to the short-term interest rate—that we control for in the VAR do not drive the empirical findings. Nevertheless, we can not fully exclude the existence of other financial shocks

²⁰The output at the industry level that the BEA constructs is only available since 2005: long past the dot-com bubble collapse. To overcome this issue, we follow Ghassibe (2021) and use the sectoral consumption constructed from NIPA's table for the information sector (NAICS 51), denoting it as Y^{IT} . We then compute the ratio between Y^{IT} and the total number of establishments in the information sector N^{IT} as a proxy for output per firm.

that may also explain the findings. Further studies that address this potential issue of observational equivalence can enhance our understanding of the source and consequences of fluctuations in the asset prices.

Having said this, asset booms caused by rational bubbles differ from booms originating from other financial shocks in terms of policy implications. For example, [Galí \(2014\)](#) demonstrates that bubbles and fundamental components react differently to monetary policy shocks. Particularly, a positive monetary policy shock increases bubbles, but depresses fundamental components. Therefore, extending our framework to conduct monetary policy analysis might offer unique insights otherwise absent in models featuring asset booms of a different origin. The joint analysis of firm dynamics and monetary policy in a bubbly versus a bubble-less economy needs further investigation.

E SVAR: Medium run restriction

E.1 The identification Issue

We now move to the discussion about the construction of the main driver of the non-fundamental component, which we refer to as the bubble shock. Note that U_t in the paper includes the vector of reduced form residuals, which are linear combinations of structural shocks:

$$U_t = B_0^{-1} \zeta_t, \quad (48)$$

where ζ_t denotes a vector of structural shocks normalized to have unit variances, $(B_0^{-1})' B_0^{-1} = \Sigma_u$, where Σ_u is the covariance matrix of the reduced form residuals. It is trivial to get an unbiased estimate $\hat{\Sigma}_u$. However, the identification issue arises because there are more parameters to be estimated in B_0^{-1} than the number of knowns contained in $\hat{\Sigma}_u$. Therefore, structural assumptions are required to overcome this identification problem. One way to understand how to impose structural assumptions is to rewrite B_0^{-1} as:

$$B_0^{-1} = A Q, \quad (49)$$

where A is a lower triangular matrix with $A A' = \Sigma_u$, e.g., the Cholesky decomposition of $\hat{\Sigma}_u$ is a natural candidate for $\hat{A}$, and Q is a orthonormal matrix such that $Q Q' = I$. Solving the identification problem boils down to find the orthonormal matrix Q such that

identification assumptions are satisfied.

We are interested in one shock, labeled as a bubble shock, hence it is sufficient to identify one column of B_0^{-1} associated with the bubble shock. To do so, we rely on the medium-run restriction. As explained above, the medium-run restriction is problematic if we fail to control for other shocks that might be, albeit unlikely, the main driver of the price-fundamental differential. In the baseline, we control for both unanticipated and anticipated productivity shocks that are, arguably, the main drivers of the business cycle.

To control for unexpected TFP shocks, following [Levchenko and Pandalai-Nayar \(2020\)](#), we include [Fernald \(2014\)](#)'s measure of utility adjusted TFP in our VAR. The first difference of this variable is widely used in the empirical literature as a measure of unexpected productivity shock, see e.g., [Garín, Lester and Sims \(2019\)](#) and [Loria, Matthes and Zhang \(2019\)](#) for recent applications, and [Ramey \(2016\)](#) for a survey.²¹ We assume that unanticipated shocks are the only shocks that affect TFP contemporaneously. With TFP ordered the first, a simple Cholesky decomposition of $\hat{\Sigma}_u$ gives us a lower triangular matrix $\hat{A}$ whose first column is associated with the TFP shocks:

$$B_0^{-1}\zeta_t = \underbrace{\begin{bmatrix} * & 0 & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ * & * & * & * & * & * \end{bmatrix}}_{\hat{A}} \begin{bmatrix} \epsilon_t^{TFP} \\ u_{2,t} \\ u_{3,t} \\ u_{4,t} \\ u_{5,t} \\ u_{6,t} \end{bmatrix} \quad (50)$$

In addition, we control for anticipated productivity shocks, i.e., news shocks. Those shocks are identified as a linear combination (Q_1) of remaining reduced form residuals, $[u_{2,t}, \dots, u_{6,t}]'$, such that it contributes the most to the cumulative sum of the square of the forecast error of TFP, i.e., $\sum_{h=0}^{H_1} (FE_{t+h|t}^{TFP})^2$. With this Q_1 , the second shock is identified as news shocks:

$$B_0^{-1}\zeta_t = \hat{A}Q_1 \begin{bmatrix} \epsilon_t^{TFP} & \epsilon_t^{news} & u_{3,t} & u_{4,t} & u_{5,t} & u_{6,t} \end{bmatrix}'. \quad (51)$$

In the next section, we explain the procedure to derive Q_1 .

²¹An alternative popular approach to identify productivity shock relies on long-run restriction ([Blanchard and Quah 1989](#) and [Galí 1999](#)), this is not suitable as we show both empirically and theoretically that bubble shocks have very persistent effect on productivity.

The last identification assumption assumes that, once controlled for unanticipated and anticipated news shocks, bubble shocks are the ones that maximize the forecast error variance decomposition of price-fundamental differential. That is, bubble shocks are identified as a linear combination (Q_2) of remaining reduced form residuals, $[u_{3,t}, u_{4,t}, u_{5,t}, u_{6,t}]'$, such that it contributes the most to the cumulative sum of the square of the forecast error of $p_t - f_t$, i.e., $\sum_{h=0}^{H_2} (FE_{t+h|t}^{p/f})^2$, where f_t is defined as in equation (20) in the paper. With such an orthonormal matrix Q_2 in place, the third shock is identified as the bubble shock:

$$B_0^{-1}\zeta_t = \hat{A}Q_1Q_2 \begin{bmatrix} \epsilon_t^{TFP} & \epsilon_t^{news} & \epsilon_t^b & u_{4,t} & u_{5,t} & u_{6,t} \end{bmatrix}'. \quad (52)$$

In what follows, we illustrate the formal procedure to derive Q_2 and we demonstrate how to control for additional shocks.

E.2 Technical Details

The starting point is to obtain impulse responses to reduced form residuals. Those can be obtained by considering our VAR(p) in companion form:

$$\mathbb{Y}_t = \mathbb{A}\mathbb{Y}_{t-1} + \mathbb{U}_t,$$

where

$$\mathbb{Y}_t \equiv \begin{bmatrix} Y_t \\ \vdots \\ Y_{t-p+1} \end{bmatrix}, \mathbb{A} \equiv \begin{bmatrix} B_1 & B_2 & \dots & B_{p-1} & B_p \\ I_K & 0 & & 0 & 0 \\ 0 & I_K & & 0 & 0 \\ \vdots & & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & I_K & 0 \end{bmatrix}, \mathbb{U}_t \equiv \begin{bmatrix} U_t \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

where K is the number of variables. Solve this equation forward:

$$\mathbb{Y}_{t+h} = \mathbb{A}^{h+1}\mathbb{Y}_{t-1} + \sum_{j=0}^h \mathbb{A}^j \mathbb{U}_{t+h-j},$$

Multiply this equation by $\mathbb{J} \equiv [I_K, 0_{K \times K(p-1)}]$ yields:

$$\begin{aligned} Y_{t+h} &= \mathbb{J}A^{h+1}Y_{t-1} + \sum_{j=0}^h \mathbb{J}A^j U_{t+h-j} \\ &= \mathbb{J}A^{h+1}Y_{t-1} + \sum_{j=0}^h \mathbb{J}A^j \mathbb{J}' \mathbb{J} U_{t+h-j} \\ &= \mathbb{J}A^{h+1}Y_{t-1} + \sum_{j=0}^h \mathbb{J}A^j \mathbb{J}' U_{t+h-j}. \end{aligned}$$

Therefore, the response of the variable $j = 1, \dots, K$ to a reduced form residual u_{kt} that occurred h periods ago, is given by:

$$\Phi_h \equiv [\phi_{jk,h}] \equiv \mathbb{J}A^h \mathbb{J}'.$$

The h -step ahead forecast error is:

$$Y_{t+h} - Y_{t+h|t} = \sum_{i=0}^{h-1} \Phi_i U_{t+h-i}.$$

Hence the MSFE at horizon h is:

$$MSFE_h = \sum_{i=0}^{h-1} \Phi_i \Sigma_u \Phi_i'.$$

Identifying News Shocks By imposing structural assumptions through an orthonormal matrix Q , the structural impulse response is given by $\Phi_h A Q$. And recall that A is the lower triangular matrix resulting from the Cholesky decomposition of Σ_u . With TFP ordered the first, the first shock is identified as unanticipated productivity shock under the assumptions that no other shocks can affect TFP contemporaneously. News shocks are identified as the linear combination (Q_1) of remaining reduced form residuals that contribute the most to the MSFE of TFP at horizons up to H_1 .

$$Q_1 = \operatorname{argmax} \frac{e_i' (\sum_{h=0}^{H_1} \Phi_h \hat{A} Q_1 e_j e_j' Q_1' \hat{A}' \Phi_h') e_i}{e_i' (\sum_{h=0}^{H_1} MSFE_h) e_i}, \quad (53)$$

s.t.

$$Q_1 \equiv \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & Q_1(2,2) & & Q_1(2,p) \\ \vdots & & \ddots & \\ 0 & Q_1(K,2) & & Q_1(K,p) \end{bmatrix}$$

$$Q_1 Q_1' = 1$$

e_i denotes selection vectors with one in the i th place and zeros elsewhere. In our empirical application, we set $i = 1$ and $j = 2$ to label the second shock as the one that maximize forecast error variance of the first variable. Note that the first column and the first row of Q_1 are specified in this way to select non-productivity shocks at the same time guarantee its orthogonality.

Identifying Bubble Shocks Once we have identified the first two shocks, we identify the bubble shocks as the linear combination (Q_2) of the remaining residuals that contribute the most to the MSFE of $p_t - f_t$ at horizons up to H_2 .

The response of $\sum_{i=0}^{\infty} \Lambda^i E_t(Y_{t+i})$ to a shock that occurred h periods ago, upon a successful selection of the matrix Q_2 , is given by:

$$\tilde{\Phi}_{h,0} = \mathbb{J} \mathbb{A}^{h-1} (1 - \Lambda \mathbb{A})^{-1} \mathbb{J}' A Q_1 Q_2$$

The response of $\sum_{i=0}^{\infty} \Lambda^i (1 - \Lambda) E_t(Y_{t+i+1})$ to a shock that occurred h periods ago is given by:

$$\tilde{\Phi}_{h,1} = (1 - \Lambda) \Lambda \mathbb{J} \mathbb{A}^h (1 - \Lambda \mathbb{A})^{-1} \mathbb{J}' A Q_1 Q_2$$

Hence, the response of the variable f_t to a shock that occurred h periods ago, is given by:

$$\tilde{\Phi}_h^f = \tilde{\Phi}_{h,1}(d) - \tilde{\Phi}_{h,0}(r),$$

where $\tilde{\Phi}_{h,1}(d)$ and $\tilde{\Phi}_{h,0}(r)$ selects, respectively, the vector of response associated with d_t and r_t . The response of $p_t - f_t$ to a shock that occurred h periods ago, is given by:

$$\tilde{\Phi}_h^{p/f} = \Phi_h(p) - \tilde{\Phi}_h^f.$$

Therefore, the MSFE of $p_t - f_t$ at horizon h is given by:

$$MSFE_h^{p/f} = \sum_{i=0}^{h-1} \tilde{\Phi}_i^{p/f} \Sigma_u (\tilde{\Phi}_i^{p/f})'.$$

We identify the bubble shocks as the linear combination (Q_2) of the remaining residuals that contribute the most to the MSFE of $p_t - f_t$ at horizons up to H_2 . Formally:

$$Q_2 = \operatorname{argmax} \frac{\sum_{h=0}^{H_2} \tilde{\Phi}_h^{p/f} e_j e_j' (\tilde{\Phi}_h^{p/f})'}{\sum_{h=0}^{H_1} MSFE_h^{p/f}}, \quad (54)$$

s.t.

$$Q_2 \equiv \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & Q_2(3,3) & & Q_2(3,p) \\ \vdots & & & \ddots & \\ 0 & 0 & Q_2(K,3) & & Q_2(K,p) \end{bmatrix}$$

$$Q_2 Q_2' = 1.$$

Controlling for other Shocks Our baseline identification strategy can be easily extended to control for more shocks. In our empirical application, as robustness checks, we control for credit supply, risk premium shocks, and shocks to the short term nominal interest rate. To do so, we include (i) a measure that includes risk premium — VIX ([Bekaert *et al.* 2013](#)), (ii) credit supply — [Gilchrist and Zakrajsek \(2012\)](#)'s excess bond premium, and (iii) the short term nominal rate into our VAR, separately to avoid the curse of dimensionality. These variables are ordered the second in the VAR system. Shocks to these control variables are identified using the classical recursive restrictions: on impact, apart from the structural shock in question, no other structural shocks affects the variable from which the structural shocks is identified. Our bubble shocks are then identified analogically to the procedure described above: as the linear combination of reduced form residuals, excluding unanticipated and anticipated TFP shocks and the additional shock that we control for, that maximize the FEVD of the price fundamental differential.

F Data

This section provides a brief overview of the data we use throughout this paper. The stock price, dividend and earning of the SP500 are taken from [Shiller \(2015\)](#), which are updated and made available on the author’s website. We take the [Gilchrist and Zakrajsek \(2012\)](#) excess bond premium (EBP henceforth) updated by [Favara *et al.* \(2016\)](#). We include [Fernald \(2014\)](#)’s utility adjusted TFP as a measure of TFP. The remaining variables are taken from FRED: the real gross domestic product per capita (real GDP), the gross private domestic investment, and the civilian unemployment rate that are seasonally adjusted, Moody’s Baa Corporate Bond Yield (to construct real interest rate r_t), CBOE Volatility Index (VIX), 3-month treasury bill rate, and GDP deflator. All nominal variables are re-scaled by GDP deflator to obtain their real values. The Business Dynamics Statistics (BDS) provide information about firms’ entry and exit rates at annual frequency from 1977 to 2016. We interpolate them into quarterly frequency, using a simple linear interpolation.

G Linking the VAR with the Model

In this section, we apply the empirical strategies adopted in the previous section using data simulated from our structural model for validating the empirical strategies and providing plausible structural interpretations to the resulting reduced-form estimates.

We simulate the time series following [Boppart *et al.* \(2018\)](#). We include three types of shocks: (1) shocks to p^b , which correspond to the bubbles shocks in the baseline VAR, (2) shocks to A_t , which correspond to the technology shocks in the baseline VAR, and (3) shocks to β (i.e., shocks to R), which corresponds to the interest rate shocks. All shocks follow AR(1) processes. Without loss of generality, we set the auto-correlations of all types of shocks to 0.5. The relative standard deviations of technology, bubble, and interest rate shocks are chosen such that they account for 25%, 60%, and 10% of the FEVD of real output at horizon 40, respectively. This choice corresponds to the main driver of the non-fundamental component justifying more than 60% of the FEVD of the real GDP at 40 quarters ahead. Our results are robust to different parameterizations of shocks processes. We add small measurement errors (with standard errors equal to 4% of the standard deviation of interest rate shocks) to each simulated data to avoid dynamic singularity.

The Decomposition of Asset Prices using VAR We treat the simulated data similarly as we treat the data in Section 5 of the paper with the same sample size (160 periods) to obtain the estimated price-fundamental differential components. We repeat this exercise 1000 times using 1000 simulated samples. Figure G.1 plots the distribution of correlations between the true bubble in the model and the estimated price-fundamental differential component of the aggregate asset price. A correlation that equals 1 indicates a perfect match. Our simulation shows that our decomposition strategy based on a linear VAR successfully captures the truth in a world described by our nonlinear structural model.

What Is the Main Driver of the Non-Fundamental Component? The following simulation exercise shows that in the model, the medium-run restriction identifies bubble shocks. We estimate the SVAR with the medium-run restriction using 1000 sample simulated from the model.

The blue dashed lines in Figure G.2 plot the effects of a positive bubble shock in the model, called the true IRFs. The red lines in Figure G.2 plot the estimated IRFs by applying our SVAR model to the simulated data. The solid red line indicates the median of the estimated IRFs from 1000 simulations. The dotted red lines represent the 90% intervals of the estimated IRFs from the 1000 simulations. The median of the estimated IRFs (out of 1000 simulations) matches the true IRFs underlying the model.

We conduct the same exercise using the data simulated from the model without bubbles. Figure G.3 reports the results. Without bubbles, the model fails to produce empirical findings presented in the paper.

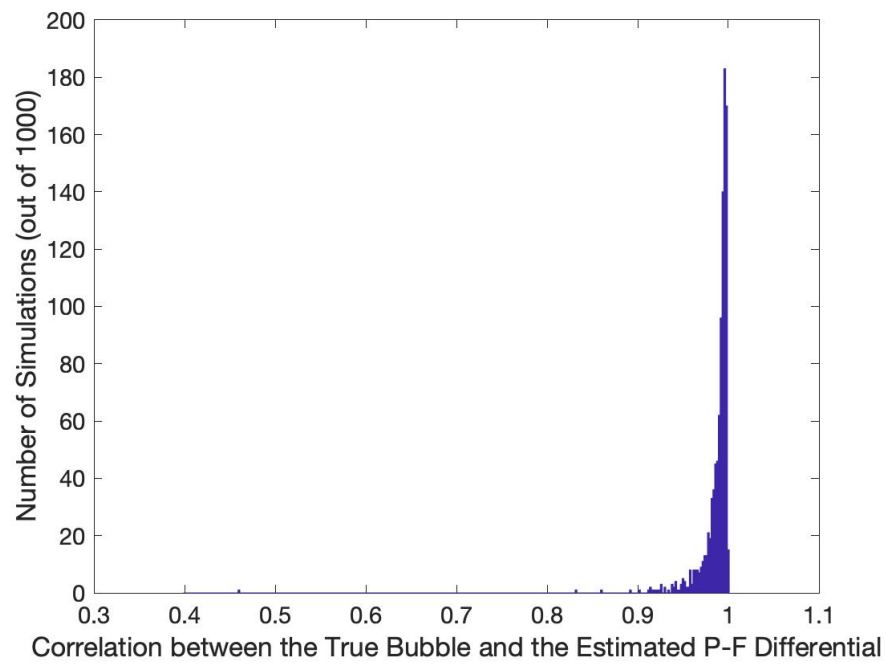


Figure G.1: This figure plots the distribution of the correlation between the true bubble and the estimated price-fundamental differential component based on 1000 simulations of the model.

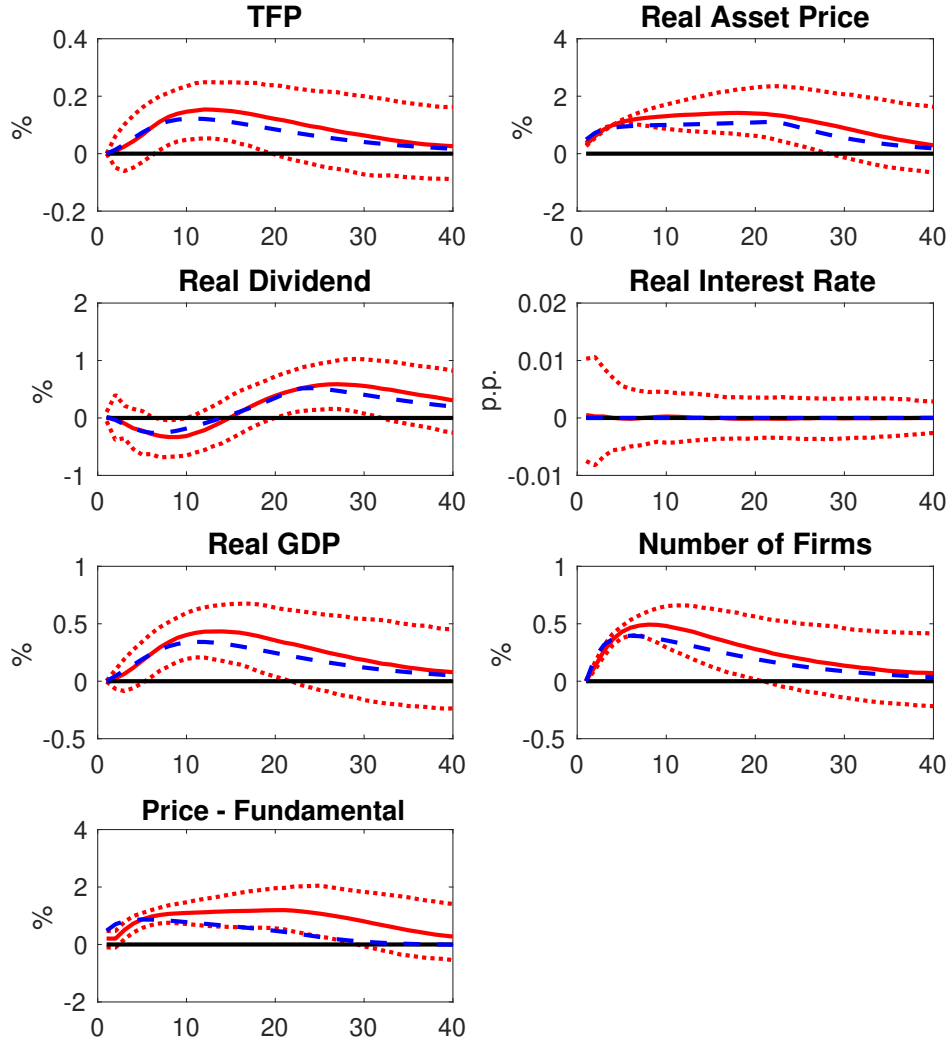


Figure G.2: This figure plots the true IRFs to a positive bubble shocks in the model (blue dashed line) together with the estimated IRFs by applying our SVAR model to the simulated data (red lines). The solid red line indicates the median of the estimated IRFs from 1000 simulations. The dotted red lines represent the 90% intervals out of the 1000 simulations. Each simulated data set consists of 160 periods—the same sample length as in the data.

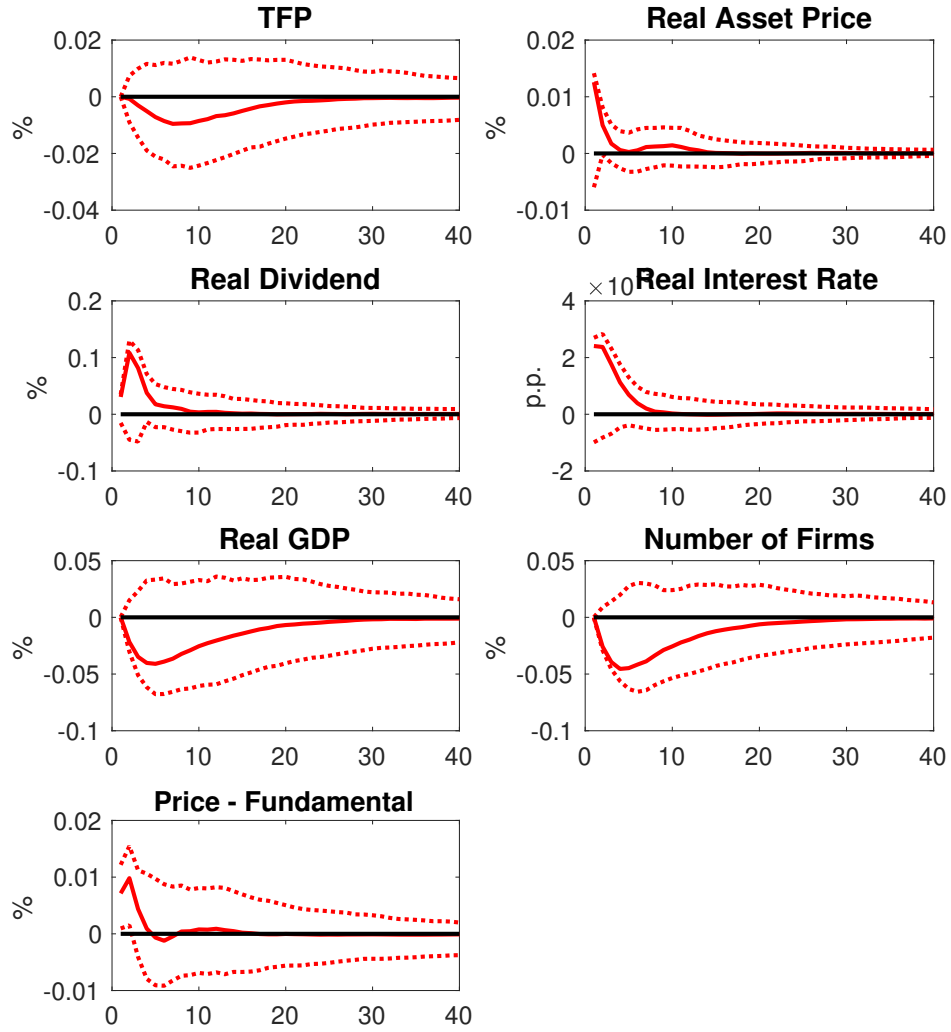


Figure G.3: This figure plots the estimated IRFs to the shocks by applying our SVAR model to data simulated from a model without bubbles. The solid red line indicates the median of the estimated IRFs from 1000 simulations. The dotted red lines represent the 90% intervals out of the 1000 simulations. Each simulated data set consists of 160 periods—the same sample length as in the data.

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