

Term Structure, Forecast Revision and the Information Channel of Monetary Policy

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Abstract

Recent studies showed that monetary policy announcements have an impact on interest rates at long horizons (10 years or more). And a contractionary monetary policy has expansionary effect on agents' expectations. These facts challenge the theories prevailing in academic and policy circles, which are based on the paradigm that monetary policy has little long-run effects and a contractionary monetary policy should depress agents' belief. In this paper, I propose a novel theory to rationalize those facts, based on the information channel of monetary policy. I consider a framework where the central bank has private information about future economic conditions. Policy actions partially reveal that information to the public, and may thus have an impact on both short- and long-term interest rates. Moreover I provide novel facts that the aforementioned responses are stronger when monetary shocks are expansionary. In light of this asymmetry, I extend the baseline model by introducing signals of uncertain precisions and ambiguity averse agent. The ability of the model to match the empirical facts is tested through a Bayesian estimation exercise. In addition to rationalize those facts, my model produces the following results. First, the information channel of monetary policy dampens the effect of monetary policy substantially. Second, the effects of monetary shocks on output and inflation are asymmetric as documented in the empirical literature. Third, introducing information asymmetry does not affect the optimal monetary policy simple rule.

Keywords: Monetary Policy, Yield Curve, Professional Forecasts, Information Asymmetry, Ambiguity, Kalman Filter

JEL Classification: E32, E43, E44, E52, E58

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“What I’ve learned at the Federal Reserve is a new language which is called ‘Fed-speak’. You soon learn to mumble with great incoherence.”

— Alan Greenspan, *Former Federal Reserve Board chairman*

1 Introduction

Motivation Before the Federal Open Market Committee (FOMC)’s September meeting in 2015, it was uncertain whether the Federal Reserve Bank (Fed) would raise the fed funds target. One side of the argument is that the U.S. domestic economy had started recovering therefore a monetary tightening was necessary to avoid future inflation. Others were concerned about the overseas economy, namely the stock market crash in China, and its consequence to the U.S. economy. Hence, the US economy might not yet be ready to exit the zero lower bound. The debate originated from the fact that private agents were uncertain about future economic condition. On September 17th 2015, the FOMC meeting took place and the Fed chair Janet Yellen announced that the committee had decided to remain the fed funds target unchanged, this was an expansionary surprise to the market¹. In response to this, the S&P500 of U.S. stock market dropped by 1.6%—the biggest drop in the month. The same experience was shared in Europe on March 10th 2016, the ECB cut its main interest rate from 0.05% to 0% and cut its bank deposit rate, from minus 0.3% to minus 0.4%. The market responded negatively, Frankfurt closed down 2.3%, Paris ended 1.7% lower and the FTSE 100 slid 1.8%.

One interpretation of previous stories is that: by observing the central bank’s expansionary action the market learns that the economy will be worse than they expected. As a result of this belief updating, the stock price dropped. These are just few out of many stories that highlight the importance of the information channel of monetary policy — agents learn the central bank’s private information by observing the central bank’s action. In the rest of the paper, I will discuss empirical facts that are puzzling in standard model but can be rationalized in a model with the presence of information channel.

Empirical Puzzles The first set of facts (**Fact 1**) describe the dynamics between yield curve and monetary policy shock. Recent empirical studies showed that monetary policy effects the long-term interest rate on impact, see for example: [Gürkaynak, Sack and Swanson \(2005b\)](#) [Hanson and Stein \(2015\)](#), [Nakamura and Steinsson \(2015\)](#), [Gertler and Karadi \(2015\)](#) and [Gilchrist](#),

¹As a fraction of agents expected a rise in fed funds rate, overall this was an expansionary surprise to the aggregate economy

López-Salido and Zakrajšek (2015)). Those papers construct monetary policy surprises using High Frequency Identification (HFI) strategy following the seminal work of Kuttner (2001).² In the baseline regression specification, I reproduce facts that are inline with the findings in the literature: in response to 1 basic point of monetary surprise, nominal interest rate with maturity of 10 years increase by 0.5 basic points on impact. The results are robust to, among other robustness checks, the use of Romer and Romer (2004)(R&R) monetary shocks as identified shocks or the forward rate of inflation indexed rate as dependent variables.

Current theoretical frameworks, namely New Keynesian (NK) models (Galí (2008)) that are widely used by central banks, are yet silent about this feature in the data. In a standard NK model, since long term rate is the weighted average of current and expected future short rate, Fact 1 can be interpreted as monetary shock has highly persistent effect to the real economy for more than 10 years. However, this interpretation requires an unreasonably high level of price stickiness and high level of inertia in monetary policy rule in a basic New Keynesian model. This paper builds a micro founded model, based on an information channel of monetary policy (Melosi (2015)) that rationalizes Fact 1 without requiring extreme calibration of parameters or time varying term premia (as argued in Hanson and Stein (2015)).

The second set of facts (**Fact 2**) describe the dynamics between private agents' expectation and monetary policy shock. Romer and Romer (2000) show that FOMC staff's internal forecasts, which are published with a lag of five years, dominate commercial forecasts. And they show that deflationary changes in monetary policy action have inflationary effects on private forecasts. Those results suggest that the Fed holds private information regarding the development of the economy and agents are learning the Fed's private information from monetary policy action. Recently, Campbell et al. (2012), Nakamura and Steinsson (2015) and Hubert (2015) revisit this idea and similar results are presented in this paper: in response to a contractionary monetary policy shock, on impact agents from survey of professional forecaster revise upwards their expectation about real GDP in the following quarters. A sign that is the opposite of what the standard NK model would predict.

The Model I build and estimate a model in which private agents' information set is nested in central bank's. I extend the basic three equations New Keynesian model by introducing: first the long term unobservable productivity trend (g_t) and second the central bank has superior information about g_t . Shock to productivity trend can be interpreted as news shock as discussed in Barsky

²The basic idea is that financial contracts contains the market's belief about future monetary actions. Thus the tick-by-tick data on Federal Funds futures and Eurodollar futures enable us to construct monetary surprises within 30 minutes around the FOMC announcements.

and Sims (2012). Thus one can interpret the assumption as the central bank has superior information about the news shocks. As a result, an expansionary (negative) monetary surprise is perceived to have two different interpretations. First it can be perceived as the Fed's endogenous response to worse than expected drop in fundamental long term trend g_t , if this is the case the private expectation about productivity trend ($E(g_t)$) decreases and consequently the market's expectation about the natural real interest rate ($E(r_t^n)$) drops. Natural real interest rate is defined as the real interest rate under flexible price, i.e. the one that prevails in the long run thus the forward rate response proportional to monetary surprise. Alternatively, an expansionary monetary surprise can be interpreted as a pure monetary shock, if this is the case it contains no information about g_t and therefore $E(g_t)$ and $E(r_t^n)$ are not affected. In my framework, since private agents cannot distinguish shock to g_t from monetary shock, optimal belief updating (Bayes' rule) requires that agents put weights on both interpretations. Thus in response to a negative monetary surprise (driven by pure monetary shock or shock to g_t), $E(g_t)$ and $E(r_t^n)$ drop hence forward rates decrease and expected real GDP in the next quarters drop as discussed previously in Fact 1 and Fact 2. The model is estimated using Bayesian method. Four main results stand out, using posterior estimates.

First, the model with asymmetric information fits well both Fact 1, Fact 2 and it is capable of reproducing Romer and Romer (2000)'s regression results. Second, information channel dampens the traditional effect of monetary policy through Euler equation because agents in the model are more pessimistic (optimistic) when they observe expansionary (contractionary) monetary surprise. The intuition is the following. While through the conventional channel a higher interest rate leads to a higher saving rate by reducing the aggregate consumption. On the other hand a positive monetary surprise is partially interpreted as the future is better than previously expected. Thus through the information channel, the over-optimistic agent that desires consumption smooth tend to increase her consumption. Third, with the presence of sticky price, information about news shocks are detrimental to welfare. Thus, it is optimal for the central bank to be fully opaque. Baeriswyl and Cornand (2010) find a similar result for mark-up shocks in a static setup. Fourth, introducing information asymmetry does not affect the optimal monetary simple rule.

The Model with Ambiguous Signals Next, I extend the baseline regressions by allowing positive and negative monetary shocks affecting the yield curve and forecast revisions differently. I show two sets of puzzling results that are novel to the literature. The first, the effect of monetary shock on the long term interest rate is more pronounced when the shock is expansionary. And the second, an expansionary monetary shock has contractionary effect on agent's expectation about real GDP that is inconsistent with the prediction of a standard New Keynesian model while

a contractionary monetary shock does not affect real GDP forecast.

Theoretically, to rationalize those facts, I extend the baseline model by introducing ambiguous signals, i.e. signals of uncertain precisions, and ambiguity averse agents. In such framework, agents react more to a bad signal (expansionary monetary surprise). The model produces the following theoretical results. First, it rationalizes the new puzzling facts introduced in this paper. Second, it predicts the asymmetric effects of monetary shocks on output and inflation as it is showed in the literature.

Section (2) discusses related literatures. Section (3) presents empirical facts. Section (4) introduces the baseline model. Section (5) estimates the model and discusses implications of the model as compared to the basic New Keynesian model. Section (6) analyzes the implication for optimal simple rule and central bank communication. Section (7) extends the model by introducing ambiguous signal(s) and ambiguity averse agents. Section (8) presents the theoretical predictions of the extended model. And section Section (9) concludes.

2 Literature Review

This paper is closely related to literature that studies the signaling effect of monetary policy. [Erceg and Levin \(2003\)](#), [Kozicki and Tinsley \(2005\)](#) and [Gürkaynak, Sack and Swanson \(2005b\)](#) study the implications of models, in which inflation target π^* is not directly observed by private agent. Those models cannot rationalize the aforementioned facts for two reasons. First, change in perceived inflation target should not affect expected real interest rate in the future, the later is close to the natural real interest rate that is independent of nominal variables. Second, those models predict a negative response of long term nominal interest rate to a monetary shock, as discussed in [Gürkaynak, Sack and Swanson \(2005b\)](#), which is inconsistent with the empirical facts that I present in this paper. [Ellingsen and Soderstrom \(2001\)](#) build a backward-looking model, in which central bank has superior information and they study dynamics between the yield curve and monetary policy when central bank holds different private information. In their framework, the monetary policy rule is not subject to exogenous shock, as a result monetary policy action fully reveals the central bank's private information to the market. In my model, agents are forward looking and due to exogenous monetary policy shock, policy action serves as a noise signal and agents in the market update their belief slowly over time. More recently, [Melosi \(2015\)](#) develops a DSGE model with dispersed information in which monetary policy rate signals to firms the central bank's view about the underlining inflationary shock (demand shock or level productivity shock). In estimating his model, the author shows that the signaling channel is important to firms' inflation expecta-

tion. However, in his framework households, the ones who trade on bond market hence determine the yield curve, are assumed to have perfect information. As a result, information channel is not present in the bond market and monetary policy shock only affects the yield curve through its conventional effect on output and inflation. Hence, such model fails to capture those facts discussed here. [Baeriswyl and Cornand \(2010\)](#) and [Tang \(2015\)](#) study the optimal monetary policy in an economy where central bank has information advantage. In contrast to those papers, this paper introduces asymmetric information about news shock and argue that private agents extract information about future economic development from policy action. This is crucial to rationalize those facts that I present in this paper. Namely the responses of yield curve and real GDP forecast to exogenous monetary shocks. Another closely related paper to mine is [Nakamura and Steinsson \(2015\)](#), the authors provide estimates of monetary non-neutrality based on the high frequency impact response of yield to monetary surprise. In addition, the authors extend their baseline model by allowing for information effect in an ad-hoc fashion: they assume an ad-hoc fraction (calibrated) of monetary surprise is perceived as originating from changing in natural real interest rate, in other words information effect arises by assumption. Independently developed from their model, this paper provides a micro founded model that fully rationalizes the belief updating process. Thanks to the micro-foundation, further extension such as the one presented in section (7) is made possible. Moreover, to the best of my knowledge, this is the first paper that documents and rationalizes the asymmetric impact of monetary surprise on yield curve and forecast revision.

The study of monetary models with imperfect information dates back to [Lucas \(1972\)](#). In his island model the author shows that money non-neutrality arises from the imperfect knowledge about those nominal disturbances. [Woodford \(2002\)](#) introduces high-order expectations into [Lucas \(1972\)](#)'s model, together with strategic complementarity [Woodford \(2002\)](#)'s framework is capable of generating large and persistent effect of a nominal disturbance. [Sims \(2003\)](#), [Mackowiak and Wiederholt \(2009\)](#) and [Mankiw and Reis \(2002\)](#) provide alternative frameworks in which the real effect of monetary shock emerges from information friction. In this paper, the information friction does not play the role of generating money non-neutrality. In fact, with the presence of information asymmetry, the real fluctuations originating from nominal disturbances are dampened. Those papers provide good substitutes for the Calvo sticky price assumed in this paper, however, similar to the standard NK model with Calvo sticky price, non of those papers is capable of generating the empirical puzzles that I aim to rationalize in this paper.

Another closely related literature is the central bank communication. A related term is "Delphic" forward guidance, which was first introduced by [Jeffrey Campbell, Chales Evans, Jonas Fisher and Alejandro Justiniano \(2012\)](#): "Delphic forward guidance publicly states a forecast of

macroeconomic performance and likely or intended monetary policy actions based on the policy-maker's potentially superior information about future macroeconomic fundamentals and its own policy goals". With full transparency, the information channel of monetary policy vanishes: had the central bank shared everything with the public, there would be no need to extract information from monetary actions. Although the Fed has become more and more transparent, its communication is yet far from being perfect. For instance the Greenbook, officially subtitled as "Economic and Financial Conditions: Current Situation and Outlook", is only available to the public with a lag of five years. The early argument against central bank transparency dates back at least to [Cukierman and Meltzer \(1986\)](#): opacity is the necessary for the central bank to be able to affect the real economy. The argument is similar to [Lucas \(1972\)](#) in the sense that full transparency makes money neutral. More recently, there are articles that argue the optimal central bank transparency depends on: i) the conduct of monetary policy; ii) the accuracy of the central bank's forecast, iii) the sources of business cycles and iv) how the social planner values output gap and inflation stabilization differently. See for examples: [Walsh \(2007\)](#), [Baeriswyl and Cornand \(2010\)](#), [Angeletos, Iovino and La'O \(2016\)](#), [Lorenzoni \(2010\)](#) and [Hahn \(2014\)](#)³. This paper contributes to this literature showing that increasing the precision of public signals about the news shocks is detrimental to welfare.

The idea that ambiguity averse agents react to signals of uncertain qualities asymmetrically dates back to [Epstein and Schneider \(2008\)](#). Based on the same ambiguity structure, [Ilut, Kehrig and Schneider \(2015\)](#) present a model in which hiring decision rules are asymmetric. [Ilut \(2012\)](#) rationalizes the uncovered interest rate parity puzzle with a model featuring information friction and signals of uncertain precisions. This paper adapts the same idea to a framework in which central bank's action plays a signaling role. And study the implications for the dynamics economic activities to monetary shocks. In another related paper, [Baqae \(2015\)](#) provide a model in which agents have imperfect information about money supply shock and the signal about monetary supply is ambiguous. In his framework, similar to [Lucas \(1972\)](#), money non-neutrality arises due to the imperfect information about nominal disturbances. Ambiguity affects the perceptions of those shocks depending on the sign of surprises. As a result money non-neutrality is asymmetric: contractionary monetary shock has bigger impact on output and inflation — the same implication is shared by the current paper. However, the channel is different. In the current paper, money non-neutrality arises from sticky price. And the asymmetric impact on output and inflation emerges because a expansionary monetary shock that sends a bad signal is trusted more than a good signal

³See also the debate on the social value of public information: [Morris and Shin \(2002\)](#), [Svensson \(2006\)](#), [Amador and Weill \(2010\)](#), [Angeletos and Pavan \(2007\)](#), and [Hellwig \(2004\)](#).

(contractionary monetary shock). This asymmetric information channel generates the asymmetric empirical puzzles discussed above — a feature that [Baqae \(2015\)](#) fails to generate. [Michelacci and Paciello \(2017\)](#) study the effects of announcement of future monetary policy in a framework with ambiguity averse household and heterogeneous financial net wealth. Policy announcements are not fully credible, and an ambiguity averse agent chooses the credibility associated with her worse case. For instance a creditor suffers from a monetary policy easing in the future therefore the ambiguity averse creditor believes such announcement more than a debtor expects. Their model predicts that the effect of forward guidance is small if wealth inequality is large.⁴ This paper differs in the following two dimensions: First, while in [Michelacci and Paciello \(2017\)](#) agents are concerned about future policy actions, in the current paper agents are mainly concerned about productivity news shock and monetary policy acts as a signal. Second, in [Michelacci and Paciello \(2017\)](#) policy announcements always confuse agent with the degree of confusion differs across agents. However in this paper, agents' posterior uncertainties always decrease (less confused) when observes a policy action with the drop in posterior uncertainty depends on the sign of monetary surprise.

This paper is also related to the literature on the information content of term structure. [Harvey \(1988\)](#), [Estrella and Hardouvelis \(1991\)](#), [Ang, Piazzesi and Wei \(2006\)](#) and [Giacomini and Rossi \(2006\)](#) show that term structure contains relevant information that are very good predictors of future economic activities, namely real output growth and consumption growth etc. This is inline with the yield curve in my model, where perceived growth rate is a key determinant of the term structure through which monetary policy has impact on long term interest rate.

If the central bank's information set does not coincide with private agent's, as a result the monetary surprise identified in the HFI literature contains the Fed's view about the fundamental that is not observed by private agents. Therefore, the HFI monetary surprise is a combination of the pure monetary shock and endogenous component that is due to information asymmetry. There is a nascent empirical literature aims to address this issue. [Lakdawala \(2016\)](#), [Campbell et al. \(2016\)](#) and [Miranda-Agrippino \(2016\)](#) regress the HFI identified monetary surprise on the information set of the central bank (Greenbook forecasts) as it is down in [Romer and Romer \(2004\)](#)⁵ to clean up the component of HFI monetary surprise that originates from asymmetric information. Alternatively, in [Francesca Loria, Carlos Montes-Galdon, Shengliang Ou and Donghai Zhang \(2017\)](#) we construct real time factors, from more than 100 macro and finance time series, that are good repre-

⁴Another related paper [Andrade et al. \(2015\)](#) study a framework where agents agree to disagree, policy announcements are perceived different across agents: can be good or bad news. Therefore with the presence of pessimists forward guidances have little impact.

⁵They use this approach to construct the conventional monetary shock: the so called Romer and Romer monetary shock

sensation of fundamental shocks yet orthogonal to monetary policy and we clean the HFI monetary surprises using those factors.

3 Empirical Facts

This section discusses empirical facts in details. Section 3.1 presents the impact effect of monetary shock on yield. Section 3.2 discusses the impact effect of monetary policy shock on real GDP forecasts.

3.1 Fact 1: Impact Effect of Monetary Policy on Yield

Following the HFI literature, the baseline regression takes the following form:

$$\Delta Y_t^h = \alpha + \beta_h \Delta MP_t + Factors + \varepsilon_{h,t}, \quad (1)$$

where, ΔY_t^h is the daily changes in nominal yield taken from [Gürkaynak, Sack and Wright \(2007\)](#). The sup-script h denotes the maturity of the yield. ΔMP_t is the daily changes in monetary policy instrument, two years nominal yield, around monetary policy decision date. The use of two years nominal yield as policy instrument is consistent with [Gilchrist, López-Salido and Zakrajšek \(2015\)](#), [Gertler and Karadi \(2015\)](#) and [Hanson and Stein \(2015\)](#), this is the common practice in the High Frequency Identification (HFI) literature to include Delphic forward guidance. It is debatable whether the daily change in monetary instrument or the change in a tighter window (ex: 30mins window around FOMC announcement) is a better proxy for monetary surprise. [Hanson and Stein \(2015\)](#) argue in favor of the daily change to allow for the market to have sufficient time to digest the new information. While [Gürkaynak, Sack and Swanson \(2005a\)](#) argue that the use of a tighter window (30 mins) around the FOMC announcement is desirable to minimize the noise. To combine the advantages of those two, I instrument the daily change of ΔMP_t by monetary surprises constructed within 30 mins window around FOMC announcement using financial future data⁶. [Gertler and Karadi \(2015\)](#) use those HFI monetary surprises, the raw data, as instruments in a Structural Vector Autoregressive model.

However, due to the asymmetric information among central bank and the private agents, the HFI monetary surprises subject to endogeneity problem, To see this, in a world with asymmetric information, a non-monetary shock that is observed by central bank but is not seen by private agents will be included in the HFI monetary surprises because central bank's reaction to this unobserved

⁶I thank Refet Gurkaynak for providing the updated 30mins window HFI monetary surprises.

shock comes out as a surprise to private agent. To address this endogeneity problem, in [Loria et al. \(2017\)](#) we construct real time factors, from more than 100 macro and finance time series, that are good representation of fundamental shocks yet orthogonal to monetary policy and we clean the HFI monetary surprises using those factors. In the current paper, I add those factors as control variables. Thus the underlining assumption is that those HFI monetary surprises, once controlled for factors, are orthogonal to $\varepsilon_{h,t}$.⁷ Intuitively, this approach is equivalent to the two steps approach: first, clean those HFI monetary surprises using factors; then use those cleaned monetary surprises as instruments. The one step approach employed in this paper has the advantage that the standard errors are free of construction error, which would arise in the two steps approach. The construction of the factors are discussed in appendix (C.1). The baseline specification controls for five factors, the same number of factors are used in [Ramey \(2016\)](#). However, the results are robust to use of different number of factors.

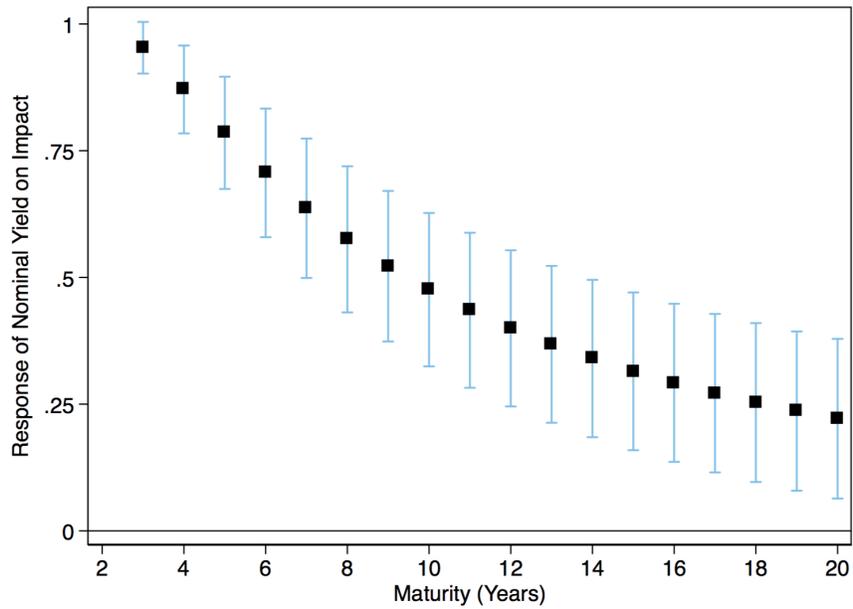
For each h , I estimate a regression of the form (1) using HFI monetary surprises as instruments for the variable of interest ΔMP_t . The sample ranges from 1995M1 to 2015M12 due to the availability of HFI monetary surprises. Figure (1) plots the estimated $\hat{\beta}$ s from regressing daily changes in nominal yield on monetary shock. Each point on the curve should be interpreted as the following: after a monetary shock that increase the monetary instrument by 1%, how much the nominal interest rate with maturity h will increase on impact. The blue lines are the 95% confidence interval constructed using standard errors that are robust to serial correlation and heteroskedasticity. As one can see, monetary surprise has persistent positive and significant effect to the entire yield curve. For instance, the nominal rate with maturity of 20 years increase by roughly 0.25 % in response to a 1% exogenous increase in monetary instrument.

The top left panel in Figure (21) plots the estimated $\hat{\beta}$ s from regressing daily changes in TIPSF on monetary surprise. As one can see, monetary surprise seems to have persistent positive and significant effect to the real forward rate until the long end (10 years ahead). I consider the TIPSF as a proxy for expected real interest rate. In NK model, the expected real interest rate in 10 years (TIPSF at horizon of 10 years) coincides with the natural real interest rate, the later depends on the growth rate of consumption (at least for the class of model that I will consider later on). Therefore, I take this fact as evidence suggests that monetary surprise affects expected growth rate of consumption in 10 years.

Asymmetric effect of monetary shock on yield curve Next, I test whether the effect of monetary shock on the yield curve is symmetric. To address this question, I estimate regressions of the

⁷Those instruments have strong first stage, with F-Statistic much bigger than the rule-of-thumb value 10.

Figure 1: Responses of Nominal Yields at Different Maturities to Monetary Shocks



Note: The square dots represent the estimated $\hat{\beta}_h$ from separated regressions: $\Delta Y_t^h = \alpha + \beta_h \Delta MP_t + Factors + \varepsilon_{h,t}$ using HFI monetary surprises as instruments for the variable of interest ΔMP_t . The blue lines are the 95% confidence interval constructed using standard errors that are robust to serial correlation and heteroskedasticity. Sample: 1995M1 to 2015M12.

following form:

$$\Delta Y_t^h = \alpha_1^h + \alpha_2^h I_{negative} + \beta_1^h \Delta MP_t + \beta_2^h \Delta MP_t \times I_{negative} + Factors + v_{h,t}, \quad (2)$$

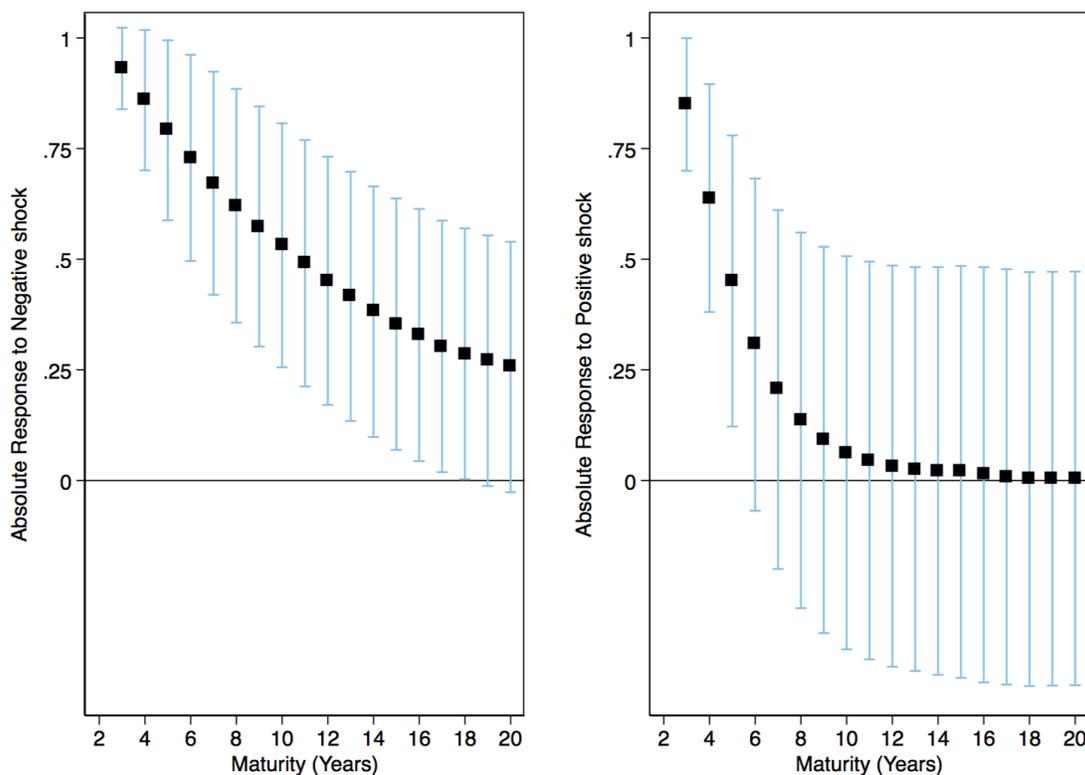
where $I_{negative}$ is the indicator variable that equals to 1 if $\Delta MP_t < 0$ and 0 otherwise. Again, ΔMP_t and $\Delta MP_t \times I_{negative}$ are instrumented by HFI monetary surprises and those interacted with $I_{negative}$.

Surprisingly, as one can see from Figure (2), while a negative (expansionary) monetary shock has significant effect on long-term yields, a positive monetary shock only affects yields with short maturities (up to 5 years).

Robustness Checks The results discussed in this section are robust to: i) excluding the recession periods, ii) number of factors, iii) controlling for the Greenbook forecasts, iv) excluding controls, v) the use of inflation indexed forward rate (TIPSF) and vi) the use of R&R monetary shocks.

Figure(17) shows the estimation results for the sample excluding the deep recession periods: namely the second half of 2008 and the first half of 2009. The asymmetry is not driven by the state of the economy. So far the number of factors included is five the same, the same number of

Figure 2: The Asymmetric Impact Effects of Monetary Shocks on Nominal Yields



Note: The square dots on the left panel represent the estimated $\hat{\beta}_1^h + \hat{\beta}_2^h$ and the square dots on the right panel represent the estimated $\hat{\beta}_1^h$, where $\hat{\beta}^h$ s are estimated from separated regressions: $\Delta Y_t^h = \alpha_1^h + \alpha_2^h I_{negative} + \beta_1^h \Delta MP_t + \beta_2^h \Delta MP_t \times I_{negative} + v_{h,t}$ using HFI monetary surprises and those interacted $I_{negative}$ with as instruments for the variable of interest ΔMP_t and $\Delta MP_t \times I_{negative}$. The blue lines are the 95% confidence interval constructed using standard errors that are robust to serial correlation and heteroskedasticity. Sample: 1995M1 to 2015M12.

factors are used in [Ramey \(2016\)](#) for her VAR analysis. Results are robust to alternative number of factors: Figure (18) plots the estimation results using eight factors — the number proposed by [McCracken and Ng \(2015\)](#) according to PCP_2 criterion. In theory, factors are good representation of fundamentals in the economy, thus are suffice to clean the endogeneity problem of HFI monetary surprise. Alternative to the use of factors, [Gertler and Karadi \(2015\)](#) and [Miranda-Agrippino \(2016\)](#) employ the Greenbook forecasts, a proxy of central bank’s information set, to handle the endogeneity problem. The results presented in this paper are robust to the inclusion of Greenbook forecasts as controls: namely the Federal Reserve Bank’s staff forecasts of real GDP, unemployment and inflation. For the estimation results, see Figure (19). Due to the availability of Greenbook forecasts, which is available to public with a lag of five years, for this robustness check the sample ranges from 1995M1 to 2011M12. Moreover, qualitatively, results are unaffected without factors

or Greenbook forecasts as controls, as it is shown in Figure (20).

So far, I have used nominal interest rates as they are widely traded thus are not subject to liquidity premium. Although this is not the case for the TIPSF, it is still interesting to see whether the evidences presented above are robust to the use of TIPSF, which is a proxy for expected real interest rate in the future. The TIPSF at long horizon corresponds, approximately, to the expected efficient real interest in the future discussed in the model. Figure (21) presents the estimation results: monetary shock effects expected real interest rate up to ten years ahead, and this is pattern is mainly driven by negative (expansionary) shocks. The sample ranges from 2004M1 to 2015M12 due to the availability of TIPSF.

Lastly, the results are robust to the use of R&R monetary shocks: an alternative measure of monetary shock that is often used in the literature.⁸ See Figure (22) for the estimation results: in baseline regressions R&R monetary shocks have big impact on yield at long horizon. And the effect is sign dependent, the impact is more pronounced when the sign of a monetary shock is negative(expansionary). The sample ranges from 1969M1 to 2007M12 due to the availability of R&R monetary shock.

3.2 Fact 2: Expected Real GDP and Monetary Policy

One interpretation of the first set of facts is that monetary surprise is perceived as news about future growth rate of consumption. The second set of facts provide evidences that are consistent with this hypothesis. Following [Romer and Romer \(2000\)](#), I estimate regressions of the following form:

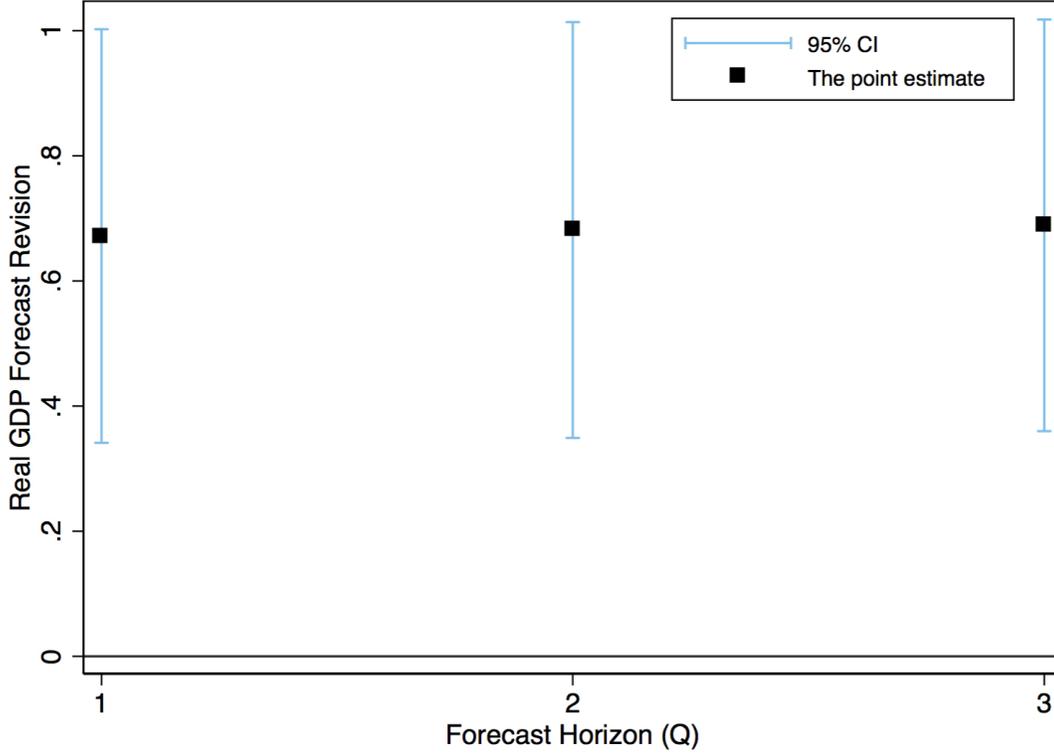
$$y_{t+j|t} - y_{t+j|t-1} = \eta + \beta_j \Delta MP_t + controls_t + v_{h,t}, \quad (3)$$

where ΔMP_t is instrumented by HFI monetary surprises. I add the following control variables: the lagged real GDP, price deflator, monetary instrument and both contemporaneous and lagged factors. According to information criteria, number of lags is chosen to be two. $y_{t+j|t}$ denotes expected log real GDP at j quarters ahead that I take from the Survey of Professional Forecasters (SPF). In order to correctly identify the effect of monetary shock on forecast revision, one needs to adjust HFI monetary surprise such that it occurred in between the current survey ($y_{t+j|t}$) and the previous survey ($y_{t+j|t-1}$). This is possible since the exact dates of SPF and FOMC meetings are publicly available.

Figure (3) plots the instantaneous effect of monetary shock on real GDP forecast revision at

⁸For this robustness check, in stead of the IV approach, I use R&R monetary shocks directly in the regressions since those are already aggregate shocks.

Figure 3: The impact effect of a monetary shock on real GDP forecast revision



Note: The square dots represent the estimated $\hat{\beta}_j$ s from separated regressions: $y_{t+j|t} - y_{t+j|t-1} = \eta + \beta_j \Delta MP_t + controls_t + v_{h,t}$. The blue lines are the 95% confidence interval constructed using standard errors that are robust to serial correlation and heteroskedasticity. Sample: 1995Q1 to 2015Q4.

1 quarter, 2 quarters and 3 quarters ahead. In response to a monetary shock that increases the monetary instrument by 100 basic points, professional forecasters revise their real GDP forecasts for the next quarters by roughly 65 basic points higher. Qualitatively, this is the opposite of what the standard NK model predicts: contractionary monetary shock should depress expected real GDP.

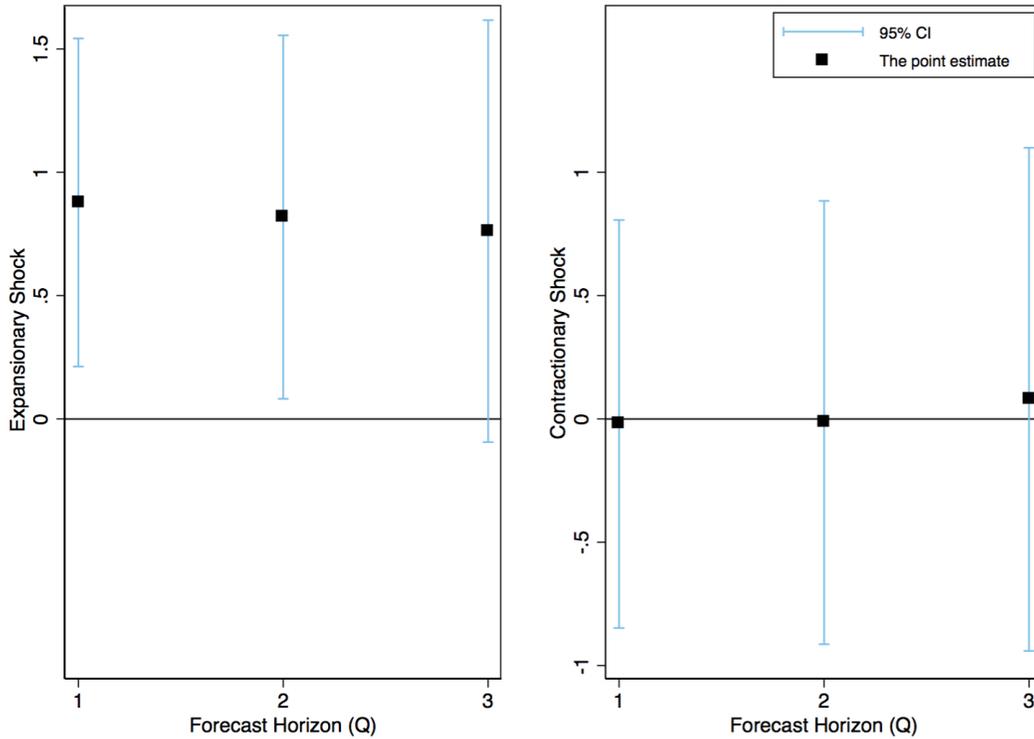
Asymmetric effect of monetary shock on real GDP forecast revision Next, I test whether the effect of monetary shock on the real GDP forecast revision is symmetric. To address this question, I estimate regressions of the following form:

$$y_{t+j|t} - y_{t+j|t-1} = \alpha_1 + \beta_1^h \Delta MP_t + \beta_2^h \Delta MP_t \times I_{negative} + controls_t + v_{h,t}.$$

Again, ΔMP_t and $\Delta MP_t \times I_{negative}$ are instrumented by HFI monetary surprises and monetary surprised interacted with $I_{negative}$. Figure (4) plots the estimation results. Similar to the case for the

yield curve, the estimated β associated with a negative (expansionary) monetary shock is positive⁹. And in contrast, a positive monetary shock does not affect real GDP forecast revision.

Figure 4: The asymmetric effect of a monetary shock on real GDP forecast revision



Note: The square dots on the left panel represent the estimated $\hat{\beta}_1$ s and the square dots on the right panel plot the estimated $\hat{\beta}_2$ s, where $\hat{\beta}$ s are estimated from separated regressions: $y_{t+j|t} - y_{t+j|t-1} = \alpha_1 + \beta_1^h \Delta MP_t + \beta_2^h \Delta MP_t \times I_{negative} + controls_t + v_{h,t}$. The blue lines are the 95% confidence interval constructed using standard errors that are robust to serial correlation and heteroskedasticity. Sample: 1995Q1 to 2015Q4.

Robustness Checks The results discussed in this section are robust to: i) excluding the recession periods, ii) number of factors, iii) controlling for the Greenbook forecasts, iv) excluding controls and and v) the use of R&R monetary shocks. See Figure (23), Figure (24), Figure (25), Figure (26) and Figure (27) for estimation results. As it is evidenced by those figures, the impact of monetary shock on real GDP forecast revision is sign dependent. While in response to a negative (expansionary) monetary shock agents revise their real GDP forecast downwards and economi-

⁹The interpretation is: in response to an expansionary monetary shock, agents revise their real GDP forecast downwards — the opposite of what the standard NK model predicts.

cally significant, a positive (contractionary) monetary shock has little impact on real GDP forecast revisions and the effect is statistically insignificant.

4 The baseline model

The basic NK model fails to rationalize the aforementioned facts. Figure (8) plots impulse responses of endogenous variables to a positive monetary shock under two sets of calibrations. The blue lines plot those under standard calibrations as in Galí (2008). The left bottom and right bottom panels plot the IRFs for expected real GDP and ten years nominal yield. The effect of monetary shock on expected real GDP growth is clearly at odds with the Fact 2 discussed above. And quantitatively, monetary policy shock has little effect on long term yield, which is inconsistent with Fact 1. The red lines plots the IRFs under extreme calibration, namely high degree of price stickiness ($\kappa = 0.01$) and high persistency of monetary policy ($\rho_m = 0.95$), in order to generate stronger response of yield to a monetary shock. With those calibrations, the response of ten year yield to a monetary shock is substantially bigger, yet still far away from what we observe in the data. The puzzling fact 2 remains to be unexplained. And more importantly, such calibration implies an unrealistic effectiveness of monetary policy as it is shown in the top left panel of Figure (8).

In this section, I build a model to rationalize the baseline (linear) facts. The model extends the basic NK model discussed in Galí (2008) with stochastic productivity trend, which is not fully observed by private agent. Shocks to the stochastic trend can be interpreted as news shock as in Barsky and Sims (2012).

4.1 Households

A representative household $j \in (0, 1)$ seeks to maximize the following utility function

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t e^{\delta_t} U(C(j)_t, N(j)_t), \text{ with } U(C(j)_t, N(j)_t) = \left[\log(C(j)_t) - \frac{N(j)_t^{1+\varphi}}{1+\varphi} \right],$$

where δ_t is the preference shock that follows:

$$\delta_t = \rho_{\delta} \delta_{t-1} + \varepsilon_t^{\delta} \text{ with } \varepsilon_t^{\delta} \sim N(0, \sigma_{\delta}^2)$$

N_t denotes labour supply and C_t is a consumption index given by:

$$C_t = \left(\int_0^1 C_{j,t}^{(\gamma-1)/\gamma} dj \right)^{\gamma/(\gamma-1)}$$

I have assumed a continuous of consumption goods $[0,1]$ with elasticity of substitution γ among them. The representative consumer faces a standard budget constraint:

$$\int_0^1 P_t(j)C_t(j)dj + Q_t B_{t+1} \leq B_t + W_t N_t + T_t$$

where, $P_t(j)$ denotes price of good j , Q_t denotes price at time t of one period bond that pays B_{t+1} at time $t + 1$, W_t the wage and T_t the lump-sum transfer including profit from firms. Solving for consumer's optimization problem leads to the following Euler equation:

$$Q_t = \beta \mathbb{E} \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \frac{P_t}{P_{t+1}} \right\}$$

where, $P_t = \left(\int_0^1 P_{j,t}^{1-\gamma} dj \right)^{1/(1-\gamma)}$ is the aggregate price index and $\Lambda_t = U_{c,t} e^{\delta t}$.

4.2 Firms

There is a continuous of firms indexed by $j \in [0, 1]$ produce differentiated goods in a monopolistic competitive market according to the following production function:

$$Y_{t,j} = e^{a_t} N_{t,j}$$

I assume that firms have access to the same technology that follows the following process:

$$\begin{aligned} a_t &= a_{t-1} + g_t + \varepsilon_t^a \text{ with } \varepsilon_t^a \sim N(0, \sigma_a^2) \\ g_t &= \rho_g g_{t-1} + (1 - \rho_g) g^* + \varepsilon_t^g \text{ with } \varepsilon_t^g \sim N(0, \sigma_g^2). \end{aligned}$$

Firms are subject to nominal rigidity *à la* Calvo (1983). Each period a fraction θ of firms cannot reset prices optimally, but choose their price according to the following indexation rule:

$$P_t(i) = P_{t-1}(i) \pi_{t-1}^\omega \pi^{1-\omega},$$

where π_{t-1} is the lagged inflation and π the steady state inflation. The fraction $1 - \theta$ of firms can

reset their prices freely and will do so optimally to maximize the following equation:

$$\mathbb{E} \left[\sum_{k=0}^{\infty} \theta^k \beta^k \frac{\Lambda_{t+k}}{\Lambda_t} (P_t(i) \Pi_{t,t+k} - MC_t) Y_{t+k}(i) \right],$$

subject to constraint:

$$Y_{t+k}(i) = \left(\frac{P_{t+k}(i) \Pi_{t,t+k}}{P_{t+k}} \right)^{-\gamma} Y_{t+k},$$

where $\Pi_{t,t+k} \equiv \prod_{s=1}^k (\pi_{t+s-1}^\omega \pi^{(1-\omega)})$.

4.3 Log-linearized equilibrium conditions

Log linearize Euler equation around steady state to get the dynamic IS equation:

$$\hat{y}_t = \mathbb{E} \hat{y}_{t+1} - [\hat{i}_t - \mathbb{E} \hat{\pi}_{t+1} - \rho_g \mathbb{E} \hat{g}_t + (\mathbb{E} \delta_{t+1} - \delta_t)]. \quad (4)$$

Since the model has a stochastic trend, to solve the model around a stationary steady state, I rescale non-stationary variable by level productivity A_t . Variable with hat denotes its deviation from steady state.

The log linearized Phillips curve is derived from firms' problem:

$$\hat{\pi}_t = \frac{\beta}{1 + \omega\beta} \mathbb{E} \hat{\pi}_{t+1} + \frac{\omega}{1 + \omega\beta} \hat{\pi}_{t-1} + \kappa \hat{y}_t + \varepsilon_t^\pi, \quad (5)$$

where $k \equiv \frac{(1-\theta\beta)(1-\theta)}{\theta(1+\omega\beta)} (\varphi + 1)$. Note that I have introduced shock to the Phillips curve: ε_t^π with mean 0 and variance σ_π .

4.4 Monetary policy

The central bank sets the interest rate follows a version of Taylor rule that keeps track of the efficient real interest rate — the one that would prevail without frictions (Wicksell (1989)). Cúrdia et al. (2015) show that the interest rate tracks the efficient rate of return fits the US. data well. Formally, Woodford (2001) show, in a framework without information friction, that such rule is optimal.¹⁰

¹⁰I do not model the monetary policy optimally because an optimal monetary policy would not allow for the existence of monetary shock. As a result, agent would learn the central bank's private information perfectly and more

Thus I assume the Taylor rule of the following form:

$$\hat{i}_t = \rho_m \hat{i}_{t-1} + (1 - \rho_m)(\phi_r r_t^e + \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t) + \varepsilon_t^m, \text{ with } \varepsilon_t^m \sim i.i.d N(0, \sigma_m^2). \quad (6)$$

The persistent component $\rho_m \hat{i}_{t-1}$ is introduced to generate persistent impacts of monetary shocks. The theoretical argument for introducing this component is the following: in a framework with the presence of output gap and inflation tradeoff, such as the current one, it is optimal for the central bank to conduct monetary policy with commitment in order to smooth the welfare loss over time. Consequently, the current policy rate depends on the previous one. Thus the lagged interest rate in the Taylor rule allows the the simple policy rule to be able to approximate the optimal monetary policy under commitment.

r_t^e denotes the efficient real interest rate.¹¹ In this framework, the efficient real interest rate is the one that would prevail if the market were perfectly competitive and if there were no information friction. To determine the efficient real interest rate, I solve the social planner's problem in this economy. The social planner makes intra-temporal consumption and labor decision according to:

$$-\frac{U_{N,t}}{U_{C,t}} = MPN_t. \quad (7)$$

The inter-temporal optimality condition under efficient allocation is:

$$R_t = \beta \mathbb{E}^e \left\{ \frac{\Lambda_t}{\Lambda_{t+1}} \right\}. \quad (8)$$

Combine (7) with (8) and log-linearize :

$$r_t^e = \mathbb{E}^e g_{t+1} - \mathbb{E}^e \Delta \delta_{t+1}, \quad (9)$$

Lastly, ε_t^m in the Taylor rule is the exogenous monetary shock. In the literature, there are two main interpretations of these monetary policy shocks. First, ε_t^m reflects shocks in FOMC's preference as committee members may prefer to response more to inflation in one day than the other. Second, ε_t^m captures measurement error in the real time data that FOMC have during the day of policy making. My preferred interpretation is the second, although the central bank aims to

importantly plotting impulse responses to a monetary shock (as the ones we have seen empirically) would not be possible.

¹¹An alternative interpretation of r_t^e in the taylor rule is that central bank reacts to its expected future economy or financial conditions, for instance the expected real GDP growth in the next period. The latter is proportional to the trend of the economy thus is similar to r_t^e .

keep track of r_t^e , due to information friction the policy rate fluctuates around the intended one ¹². What's important here is to the assumption that ε_t^m is exogenous to policy decision and monetary shocks are not observed by private agents. With the presence of unobserved monetary shock, central bank's action does not provide perfect information regarding the unknown g_t therefore agent's learning is persistent.

4.5 Belief updating and solution of the model

Private agents make decision under an uncertain environment. Namely, they do not observe the productivity trend g_t perfectly and cannot distinguish trend shocks (ε_t^g) from level productivity shocks (ε_t^a). Moreover monetary shock (ε_t^m) is not observed by agents. Since the trend is relevant for their optimal decisions, agents will infer g_t from observable variables. Denote agents' estimate of g_t by $g_{t|t} \equiv E(g_t|Z^t)$, where Z^t is the history of variables that are relevant for inference about g_t that agents observe up to time t.

I assume that agents understand the model and know the distributions of shocks as well as parameters of the model. To update $g_{t|t}$, agents use up to date history of $Z_t = [a_t \hat{i}_t s_t]$ those are: the level of productivity, the policy rate and additional private signals about g_t summarized in s_t :

$$s_t = g_t + \varepsilon_t^p \text{ with } \varepsilon_t^p \sim i.i.d N(0, \sigma_p^2). \quad (10)$$

s_t is introduced for two reasons. First, it is more realistic to assume, apart from monetary action and level productivity, agents receive additional information about future development of the economy. For example by reading news papers. Second, with the introduction of s_t the NK model with perfect information is nested in this model by setting σ_p equal to zero. Thus, when the model is estimated, one way to assess whether the information channel of monetary policy is empirically relevant or not is to check if σ_p is different from zero.

It worths to emphasis that the realized short run interest rate i_t provides information about the underlining trend g_t because agents know the central bank's reaction function. Moreover, due to the fact that agents do not observe monetary shocks, they cannot infer g_t perfectly from monetary actions.

The signal extraction problem of agents is subject to simultaneity problem. The monetary policy signal responses to endogenous variables $\hat{\pi}_t, \hat{y}_t$, and those are determined based on agents posterior belief about g_t , which in turn depends on monetary policy. [Svensson and Woodford \(2004\)](#) provide a solution for optimal filtering under these settings.

¹²That is $\varepsilon_t^m \equiv (1 - \rho_m)\rho_g(g_t - \mathbb{E}^{Fed}g_t)$.

The model's solution is derived in Appendix (A), in the model agent updates her belief about the unknown process \hat{g}_t and ε_t^m using kalman filter. And naturally $g_{t|t}$ and $\varepsilon_{t|t}^m$ are state variables. Define $X_t \equiv \left(\varepsilon_t^m \ \hat{g}_t \ \hat{\delta}_t \ \varepsilon_t^\pi \ \hat{i}_{t-1} \ \hat{\pi}_{t-1} \ \varepsilon_{t|t}^m \ \hat{g}_{t|t} \right)'$, $U_t \equiv \left(\varepsilon_t^s \ \varepsilon_t^\delta \ \varepsilon_t^\pi \ \varepsilon_t^m \ \varepsilon_t^a \ \varepsilon_t^p \right)'$ and $X_t^f \equiv \left(\hat{y}_t \ \pi_t \right)'$ then the state space representation of the model's solution is:

$$X_{t+1} = AX_t + BU_{t+1}, \quad (11)$$

$$X^f = FX_t, \quad (12)$$

with matrix A, B and F specified in Appendix (A).

4.6 Term structure

This section writes down term structure implied by the model. Following [Bekaert, Cho and Moreno \(2010\)](#) and [Nimark \(2008\)](#), yields of different maturities are derived based on next four equations together with the assumption that shocks are normal. The first equation characterizes the short run yield, which is basically the monetary policy rule:

$$i_t = M_I X_t \quad (13)$$

The second equation relates yields at different maturities with corresponding prices:

$$i_t^n = -\frac{1}{n} \log(P_t^n) \quad (14)$$

In the third equation, I assume that there is no arbitrage condition:

$$P_t^{n+1} = E_t(M_{t+1} P_{t+1}^n) \quad (15)$$

The fourth equation derives stochastic discount factor from consumer's optimization problem:

$$M_{t+1}^n = \beta \frac{\Lambda_{t+1} P_t}{\Lambda_t P_{t+1}} \quad (16)$$

Combining these equations one can write the yields in terms of state variable X_t as¹³:

¹³Find in Appendix (A) for the detailed derivations.

$$\begin{bmatrix} i_t \\ i_t^2 \\ \cdot \\ \cdot \\ i_t^n \end{bmatrix} = \begin{bmatrix} -A_1 \\ -\frac{1}{2}A_2 \\ \cdot \\ \cdot \\ -\frac{1}{n}A_n \end{bmatrix} + \begin{bmatrix} -B_1 \\ -\frac{1}{2}B_2 \\ \cdot \\ \cdot \\ -\frac{1}{n}B_n \end{bmatrix} X_t \quad (17)$$

where i_t^n denotes the yield with maturity n at time t , matrix A_n and B_n are derived recursively in Appendix (A). There are three key features that are absent in the previous literature. First in this model agents' belief about trend $\hat{g}_{t|t}$ enters as a state variable and therefore affects the whole yield curve. Second, monetary policy acts as one of the signals consequently monetary surprises affect the long end of the yield through their effects on $\hat{g}_{t|t}$ that standard model cannot capture. And third, if the process for g_t is highly persistent, which is verified in our estimation exercise later on, current shocks would affect the yield curve at all horizon. This is a feature that [Nimark \(2008\)](#) fails to capture. In his model, 87% of volatility in 1 year yield is left to be explained by measurement error.

5 Model Estimation and Results

I estimate the model by Bayesian method using quarterly data from the U.S. ranging from the first quarter of 1982 to the last quarter of 2016. Real GDP per capita, inflation, nominal yields with maturities of two, five and ten years are included in the measurement equations, see Appendix (D) for detailed data descriptions.

Identification It is well known that many DSGE models subject to local identification issue, see [Canova and Sala \(2009\)](#) for potential reasons. [Iskrev \(2010\)](#) and [Komunjer and Ng \(2011\)](#) provide algorithms to check for identification prior to estimation. It is a standard practice in the literature that unidentified parameters are then calibrated. I follow [Komunjer and Ng \(2011\)](#)'s procedure and all parameters are verified to be locally identified.

Priors and Posteriors Tables (1) and (5) report the priors and posterior estimates. Priors are reported in the first three columns. They are taken from the literature, see for example [An and Schorfheide \(2007\)](#). I estimate posteriors by Random Walk Metropolis-Hasting algorithm. Posterior means and standard errors for both the model with asymmetric information and the one with

perfect information (in which σ_p is imposed to be zero) are reported. The key parameters that drive the main results are reported in Table (1): variances of monetary, level productivity, trend and private signal shocks — those are not only a measure of volatility but also a measure of precision of signals. In the model with asymmetric information, the posterior mean of σ_p — the noise of private signal is much larger than volatility of monetary shock (σ_m). This suggests that information is indeed not perfect. In addition, the fact that σ_a , another source of private information, is relatively big determines the magnitude of information channel of monetary policy.

Table (5) reports the posterior estimates of other parameters. With few exceptions, estimated parameters between a model with asymmetric information and a model with perfect information are similar. The results discussed below are not driven by those differences.

Table 1: Prior and Posterior: key parameters

	Priors			Asymmetric Information		Perfect Information	
	Mean	s.d	Distribution	Mean	s.d	Mean	s.d
σ_m	2×10^{-3}	4	InvGamma	1.2×10^{-3}	0.8×10^{-4}	1.7×10^{-3}	1.2×10^{-4}
σ_a	2×10^{-3}	4	InvGamma	6.7×10^{-3}	4.9×10^{-4}	7.1×10^{-3}	4.1×10^{-4}
σ_g	2×10^{-3}	4	InvGamma	1.6×10^{-3}	0.1×10^{-3}	1.6×10^{-3}	0.1×10^{-3}
σ_p	2×10^{-3}	4	InvGamma	1.9×10^{-2}	0.9×10^{-2}	—	—

Note: Posterior means and standard deviations are estimated by Random Walk Metropolis-Hasting algorithm

Rationalizing Fact 1: the Impact Effects of Monetary Shocks on Long-term Yield Figure (5) plots one of the main facts that I aim to capture: the impact effects of monetary shocks on nominal yields. Those should be compared to their empirical counterpart: Figure (1). The circles and intervals are the mean estimates and 95% confidence intervals constructed based on posterior parameters' distributions. The blue lines plot the predictions of the model under asymmetric information. The red ones plot those for the perfect information case: all parameters are set to be the same as for the asymmetric case except for the volatility for private signal σ^p , which is imposed to be zero. This choice of comparison is made to ensure that the information channel is the key

mechanism driving success of the model. ¹⁴

As one can see, while the model with perfect information fails to reproduce the empirical facts, the model with information asymmetry fits the fact very well. As observed in the data, the model predicts that on impact nominal yields at all horizons response to monetary surprises positively and significantly. In the model, private agents cannot distinguish a monetary shock from central bank's endogenous response to the bank's information about the news shock. And agents know that the monetary policy responses positively to a news shock. As a result a positive monetary surprise that originates from a pure positive monetary shock is partially interpreted as the central bank has received good news about future development of the economy, hence the perceived natural real interest rate increases. Notice that the natural real interest rate is highly persistent ($\rho_g = 0.98$) in this economy, a magnitude that is shared in the long run risk literature (Bansal and Yaron (2004)), therefore expected natural real interest rate in the far future also increases. Consequently, the long term yield, which is the weighted average of current and expected future policy rate, increases in response to a positive monetary surprise.

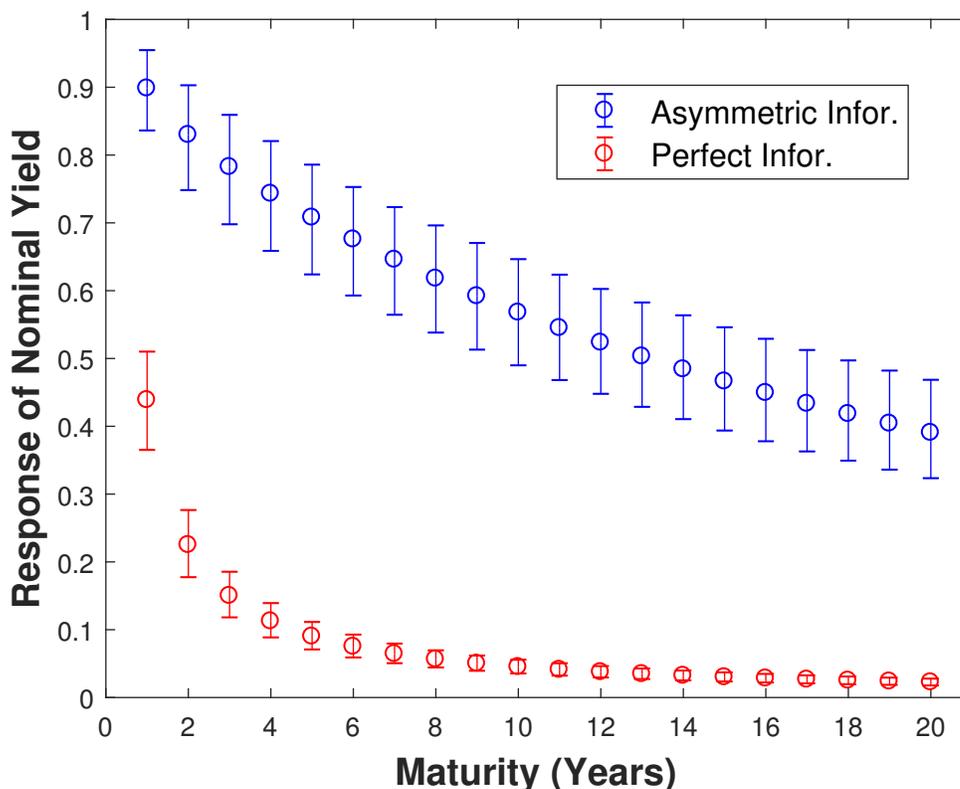
Rationalizing Fact 2: the Impact Effects of Monetary Shocks on Real GDP Forecast Revisions

Figure (6) shows the second fact that I aim to rationalize: the impact effects of monetary shocks on real GDP forecast revisions. Those should be compared to their empirical counterpart: Figure (3). Again, the figure plots the mean estimates and 95% confidence intervals both for the model with information asymmetry (in blue) and with perfect information (in red). In contrast to the prediction of standard NK model (the red lines), the model with asymmetric information fits the data well. In response to a positive monetary shock agents, both in the model and data, revise upward their forecasts of log real GDP in the next quarters. This is due to the information channel of monetary policy. A positive monetary shock is partly interpreted as the arrival of a positive news shock, as a result agents become more optimistic.

The Effects of Monetary Shocks: Asymmetric Information v.s Perfect Information Figure (9) compares the IRFs of endogenous variables to a positive monetary shock in the model with asymmetric information with those in the model with perfect information. The solid and dashed lines are the mean estimates and 95% confidence interval of IRFs of the models constructed based on the posterior parameters' distributions. The ones in blue plot the predictions of the model under asymmetric information. The red ones plot those for the perfect information case: all parameters are set to be the same as for the asymmetric case except for the volatility for private signal σ^p ,

¹⁴Alternatively, one can compare the predictions of the two models based on separate parameters estimates. The results are unchanged.

Figure 5: Prediction of the Model: the Impact Effects of Monetary Shocks on Nominal Yields



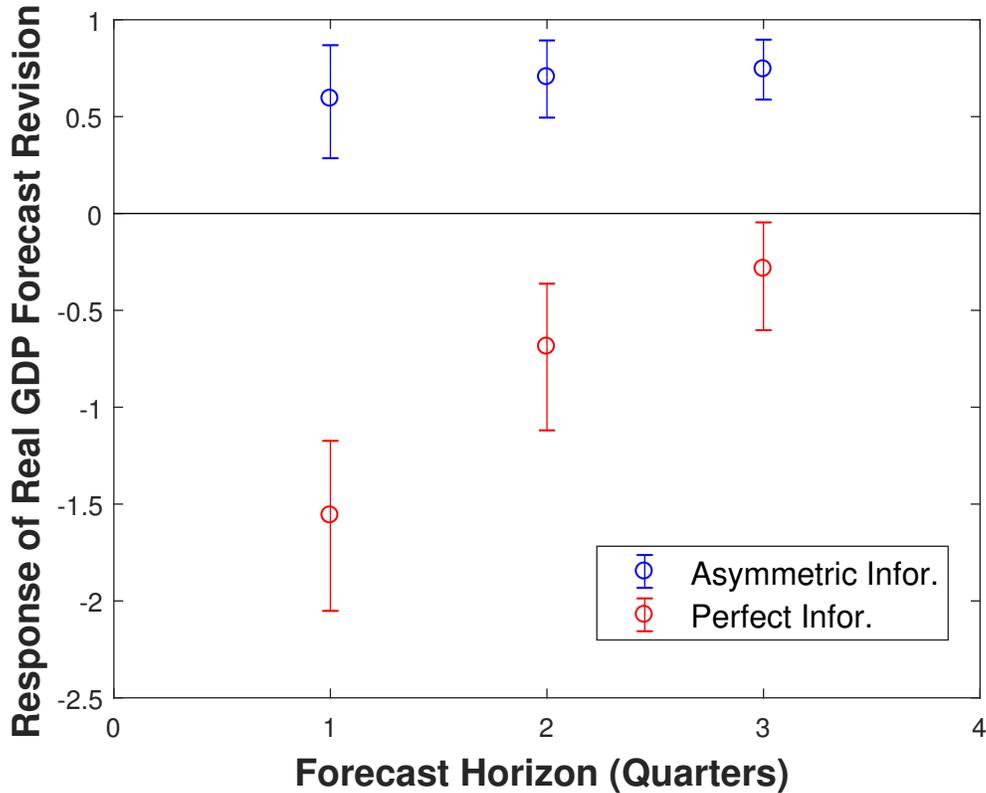
Note: This figure depicts the impact effects of monetary shocks on nominal yield at different maturities predicted by the models. The circles and intervals are the mean estimates and 95% confidence intervals constructed based on posterior parameters' distributions. The blue lines plot the predictions of the model under asymmetric information. The red ones plot those for the perfect information case: all parameters are set to be the same as for the asymmetric case except for the volatility for private signal σ^p , which is imposed to be zero.

which is imposed to be zero. The labels m , Int , $EOutput$, $EInf$ and $Y5$ denote monetary shock, the policy rate, expected log real output at one quarter ahead, expected inflation at one quarter ahead and five years nominal yield respectively.

The bottom left panel and bottom right panel confirm the previous findings (on impact): while the model with asymmetric information reproduces both Fact 1 and Fact 2 as discussed above, the model with perfect information fails to match the data. Moreover, those impulse responses functions suggest that agents misinterpret a monetary surprise at the beginning and only fully learn the truth after roughly two years.

More interesting results are shown on the top left and middle panels. As compared to the case with perfect information, the effects of monetary policy shock to output and inflation in the model

Figure 6: Prediction of the Model: the Impact Effects of Monetary Shocks on Real GDP Forecast Revisions



Note: This figure depicts the impact effects of monetary shocks on real GDP forecast revision at different horizons predicted by the models. The circles and intervals are the mean estimates and 95% confidence intervals constructed based on posterior parameters' distributions. The blue lines plot the predictions of the model under asymmetric information. The red ones plot those for the perfect information case: all parameters are set to be the same as for the asymmetric case except for the volatility for private signal σ^p , which is imposed to be zero.

with asymmetric information are more silent. Unlike in the perfect information case, in which monetary policy affects output only through consumer's Euler equation and inflation adjusts according to the Phillips curve, with asymmetric information the information channel of monetary policy emerges. A contractionary monetary policy is perceived as a good news shock, thus agents become more optimistic and as a result they consume more. The information channel of monetary policy dampens the traditional channel, as a result monetary policy shock in the model with asymmetric information is less disturbing. Moreover, in contrast to a model with perfect information, with asymmetric information the realized GDP path does not coincides with the expected GDP.

The analysis in Figure (9) are conducted based on parameters that are estimated using the

model with asymmetric information. While it is useful to cleanly pin down the information channel of monetary shock on economic activities, one cannot answer the question whether ignoring information asymmetry would lead to biased estimates for the effectiveness of monetary shocks. To address this question, Figure (10) depicts IRFs for both the model with asymmetric information (in blue) and perfect information (in red) based on separately estimated parameters. As one can see, to the extent that the true model features information asymmetry ($\sigma^P > 0$), estimating a model with perfect or symmetric information ($\sigma^P = 0$) would lead to significantly biased estimates regarding the effectiveness of monetary shocks on output and inflation.

Replicating Romer and Romer (2000) The previous analysis show that the estimated degree of information asymmetry is capable of rationalizing the aforementioned empirical puzzles. Yet we have not discussed whether the implied information asymmetry is reasonable or not. The fact that the model implied responses of real GDP forecast revisions to monetary shocks (Figure (6)) matches the empirical Fact 2 suggests that the underlining information asymmetry is reasonable. In the next exercise, I provide additional evidence.

Romer and Romer (2000) show that FOMC staff's internal forecasts dominate commercial forecasts based on estimation of the following regressions:

$$y_{t+h} = \alpha + \beta^P E_t^P(y_{t+h}) + \beta^F E_t^F(y_{t+h}) + e_t, \quad (18)$$

where $E_t^P(y_{t+h})$ denotes commercial forecasts (private agent's forecasts) at time t about variable y at h horizons ahead and $E_t^F(y_{t+h})$ denotes the Fed's internal forecasts. They do the exercise both for real GDP forecast and inflation forecast. The null hypothesis is whether $\beta^F = \beta^P$. Under the null both private and the Fed's internal forecasts are equally precise. With $\beta^F > \beta^P$ meaning the Fed's information is superior than private forecasts. Estimation results are reported in their Table (3) and Table (5): overall $\hat{\beta}^P$ is not statistically different from zero and $\hat{\beta}^F$ is close to one. Those results suggest that the Fed has superior information about future inflation and real GDP and for the predictability of those variables it is suffice to use solely the Fed's internal forecasts as predictors.

Using the structure model in this paper, I replicate their empirical results using the estimated parameters. I simulate the model for 10 000 times for 100 periods each (the same sample size as in Romer and Romer (2000) baseline regressions using Blue Chip survey data). In each simulation, I construct the realized variables y_{t+h} : real GDP and Inflation; the private agent's forecasts of the corresponding variables at different horizons h denoted as $E_t^P(y_{t+h})$; and the central bank's forecasts denoted as $E_t^F(y_{t+h})$. For each simulation, I estimate the regressions of the form (18). The mean estimates are reported in Table (2), and the standard deviations are reported in parentheses.

As one can see, the model successfully replicates [Romer and Romer \(2000\)](#)'s empirical results not only qualitatively but also quantitatively. Note that the forecasts data were not used in estimation, I interpret this as evidence suggesting that the implied information asymmetry in the model is realistic.

Table 2: Model Simulation: Romer and Romer (2000) Regressions

$$y_{t+h} = \alpha + \beta^P E_t^P(y_{t+h}) + \beta^F E_t^F(y_{t+h}) + e_t$$

Forecast Horizon h	Real GDP		Inflation	
	$\hat{\beta}^P$	$\hat{\beta}^F$	$\hat{\beta}^P$	$\hat{\beta}^F$
1 Quarter	-0.01 (0.10)	1.00 (0.13)	-0.01 (0.20)	1.00 (0.23)
2 Quarters	-0.02 (0.24)	1.00 (0.26)	-0.02 (0.34)	1.01 (0.36)
3 Quarters	-0.03 (0.44)	1.00 (0.40)	-0.03 (0.46)	1.01 (0.47)
4 Quarters	-0.02 (0.71)	0.99 (0.57)	-0.05 (0.60)	1.01 (0.58)

Notes: I simulate the model for 10 000 times for 100 periods each. In each simulation, I construct the realized variables y_{t+h} : real GDP and Inflation; the private agent's forecasts of the corresponding variables at different horizons h denoted as $E_t^P(y_{t+h})$; and the central bank's forecasts denoted as $E_t^F(y_{t+h})$. For each simulation, I estimate [Romer and Romer \(2000\)](#)'s regressions: $y_{t+h} = \alpha + \beta^P E_t^P(y_{t+h}) + \beta^F E_t^F(y_{t+h}) + e_t$. The mean estimates are reported in the table, and the standard deviations are reported in parentheses.

6 Welfare Analysis and Monetary Policy

This section discusses the optimal central bank communication and optimal monetary policy simple rule under the current framework.

The Welfare Loss Function The welfare loss function can be derived as the second order approximation of a household's welfare¹⁵:

$$W = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[(1 + \varphi) \hat{y}_t^2 + \frac{\varepsilon}{\lambda} \hat{\pi}_t^2 \right], \quad (19)$$

where the welfare loss W is expressed in terms of the consumption loss as a fraction of steady state consumption. Note that the deep parameters φ , ε and λ , those that characterize the relative importance between the variances of output gap and inflation, were not estimated due to identification issue. The analysis conducted below are based on the same parameterization as in Galí (2008): $\varphi = 1$, $\varepsilon = 6$, $\lambda = 0.17$.

Central Bank Communication Given the superior information that the central bank holds about the news shock, a natural question arises: would it be optimal for the central bank to release its private information? With full central bank transparency, the information channel of monetary policy would vanish.

Recall that, previously, I have assumed that the private agent observes a private signal about g_t :

$$s_t = g_t + \varepsilon_t^p \text{ with } \varepsilon_t^p \sim i.i.d N(0, \sigma_p^2).$$

If σ_p^2 equals to zero, the information would be perfect. Following Baeriswyl and Cornand (2010), the central bank's communication policy can be measured by σ_p^2 . A fully transparent central bank would release everything and therefore the private signal, which includes information released by central bank, would be perfect, i.e. $\sigma_p^2 = 0$. In the other extreme with a fully opaque central bank, the one that is assumed in this paper, the noise of the private signal would remain to be the estimate reported in Table (1): $1.9 (\times 10^{-2})$.

To see whether central bank transparency is beneficial or detrimental to welfare, I calculate the welfare loss (measured as a fraction of steady state consumption) for different values of $\sigma_p^2 = 0$ that ranges from 0 to 1.9. Figure (11) plots the result. The vertical axis denotes the welfare loss and the horizontal axis denotes the degree of opacity measured by σ_p^2 . Note that degree of opacity equal to zero corresponds to a fully transparent central bank and the other extreme corresponds to fully opaque one. As it is shown in the figure, opacity about the news shock improve welfare. The intuition behind is the following. In a NK framework with sticky price, a positive news shock today would lead to a boom in aggregate consumption due to individual consumption smoothing

¹⁵See Galí (2008) for detailed derivations.

decision. But current technology has not yet increased, therefore the output gap will be positive. In addition, inflation arises. Both volatility in output gap and inflation is detrimental to welfare, therefore, the representative agent would be better off if the information about the increase in technology in the future can be hidden from them.

Optimal Simple Rule I will now discuss the optimal simple monetary policy rule given the parameters' estimates. The optimal simple rule is the solution to the following problem:

$$\min_{\rho_m, \phi_r, \phi_\pi, \phi_y} W = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[(1 + \varphi) \hat{y}_t^2 + \frac{\varepsilon}{\lambda} \hat{\pi}_t^2 \right],$$

Subject to the dynamics of the economy. Recall that the control variables are the parameters (weights) in the simple Taylor rule:

$$\hat{i}_t = \rho_m \hat{i}_{t-1} + (1 - \rho_m) (\phi_r r_t^e + \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t) + \varepsilon_t^m.$$

Table (3) reports the optimal simple rule under both the asymmetric information and perfect information model. Both are calculated based on the medians of the posterior distributions of parameters estimated using the asymmetric information model, with the exception of σ_p is imposed to be zero for the case with perfect information. The optimal simple rule is nearly unchanged when information asymmetry is introduced. It is optimal to response to the lagged interested rate due to commitment. It is efficient to keep track of the efficient real rate. Woodford (2001) show that the optimal taylor rule is the one that response to the natural real interest rate one to one, the same result carry over to the current setup in which the central bank has superior information regarding the news shock. In a standard NK model with perfect information, it is optimal for the policy rate to response aggressively to inflation, the same result hold for the model with asymmetric information. Moreover, even though the both models feature a tradeoff between stabilization of output gap and inflation, the optimal weight on output gap is zero. This is the case because the response to efficient real rates already handles the stabilization of output gap.

The fifth column reports the welfare loss, as fraction of steady state consumption, under optimal simple rule of the corresponding models. The welfare loss in the model with perfect information is 1.09%, bigger than the one associated with asymmetric information model (1.04%). This confirms the optimal communication conducted above: revealing information is detrimental to welfare even under the optimal simple rule. One interesting question is: if the central bank conduct the optimal simple rule ignoring the information asymmetry, how big is the welfare loss? The last column provides an answer, it reports the welfare loss associated with the optimal simple rule derived in

the model with perfect information. If the true model is the one with asymmetric information, a central bank that ignores this feature would commit to a welfare loss of 1.05%: 0.01 percentage point higher than the one associated with the optimal simple rule.

Table 3: The Optimal Simple Rule

	ρ_m	ϕ_r	ϕ_π	ϕ_y	Welfare Loss	
					Under optimal rule	Ignorant rule
Asymmetric Information	0.65	0.98	3.70	0	1.04%	1.05 %
Perfect Information	0.57	1.00	3.72	0	1.09%	1.09%

Notes: this table reports the optimal simple rule under both the asymmetric information and perfect information model. Both are calculated based on the medians of the posterior distributions of parameters estimated using the asymmetric information model, with the exception of σ_p is imposed to be zero for the case with perfect information.

Figure (12) provides a visual representation of Table (3). Each panel plots the welfare loss by varying the parameter of interest with the rest policy coefficients are fixed at at the optimal simple rule computed according to the asymmetric information model, i.e. the first raw in Table (3). The blue (red) lines plot the results for the model with asymmetric (perfect) information. As one can see, for all of those four parameters, the minimum is achieved at the parameter value reported in Table (3). Moreover, consistent with the optimal transparency policy analysis conducted above, the red lines always lay above the blue line suggesting that welfare loss is bigger when information is perfect.

In sum despite of the minor quantitative differences in the optimal ϕ_m and ϕ_r , the policy implications for the design of optimal simple monetary rule is unaffected by the introduction of information asymmetry: the lessons learned from the basic NK model remain valid.

7 A model with ambiguity averse agents

The previous two sections present a baseline model that rationalizes the baseline (linear) facts. However, the model predicts symmetric effects of monetary policy on yield curve and forecast revision, which are inconsistent with the novel (asymmetry) facts I introduced in section (3), namely Figure (2) and Figure (4). In order to generate the asymmetry in the same family of model, I introduce the following additional assumptions: the first the volatilities of shocks are uncertain and

the second agents are ambiguity averse. For illustration, I show the extension in which monetary policy as signal is ambiguous i.e. they do not know the exact distribution of monetary shock in the sense that the volatility of monetary shock is unknown. But they know it lies in between $[\underline{\sigma}_m, \bar{\sigma}_m]$. However, results are robust if one or many of the following volatilities are uncertain: level productivity shock, trend productivity shock, private signal shock and monetary shock. Since, ambiguity in those shocks lead to an ambiguous Kalman gain in agent's belief updating equation. The latter is crucial for rationalizing the asymmetric facts.

Household A representative household $j \in [0, 1]$ who is ambiguity averse solves the following optimization problem:

$$\max_{C(j)_t, N(j)_t} \min_{\sigma_m^t \in \Gamma^t} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t e^{\delta t} \left[\log C(j)_t - \frac{N(j)_t^{1+\varphi}}{1+\varphi} \right].$$

I have assumed that agents have multiple priors about the quality of monetary policy signal. This Knightian uncertainty is axiomatized by [Gilboa and Schmeidler \(1989\)](#) and [Epstein and Schneider \(2003\)](#). Due to ambiguity, agents believe that σ_m can potentially be time varying. σ_m^t denotes the perceived variance of monetary shock up to time t . $\Gamma^t = \underbrace{\{\Gamma \times \dots \times \Gamma\}}_{t \text{ times}}$ with $\Gamma = [\underline{\sigma}_m, \bar{\sigma}_m]$. Ambiguity averse agents behave according to their worse case belief, denoted as $\tilde{\mathbb{E}}_t$. Therefore, one can simplify household's optimization problem as:

$$\max_{C(j)_t, N(j)_t} \tilde{\mathbb{E}}_t \sum_{t=0}^{\infty} \beta^t e^{\delta t} \left[\log C(j)_t - \frac{N(j)_t^{1+\varphi}}{1+\varphi} \right]$$

with the choice of prior σ_m^t that enters in belief $\tilde{\mathbb{E}}_t$ remains to be determined. Solving for consumer's optimization problem, I find the following Euler equation:

$$Q_t = \beta \tilde{\mathbb{E}}_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \frac{P_t}{P_{t+1}} \right\}$$

Log-linearized equilibrium conditions The model is solved by taking the following steps: i) guess a σ_m^t that $\min_{\sigma_m^t \in \Gamma^t} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t e^{\delta t} U(c_t, n_t)$ ii) solve the loglinearized model under the conjectured worse case belief iii) verify the initial guesses.

Given an initial guess of utility minimizing σ_m^t , the log linearized Euler equation around steady

state¹⁶ to get the dynamic IS equation:

$$\hat{y}_t = \tilde{\mathbb{E}}_t \hat{y}_{t+1} - [\hat{i}_t - \tilde{\mathbb{E}}_t \hat{\pi}_{t+1} - \rho_g \tilde{\mathbb{E}}_t \hat{g}_t + (\tilde{\mathbb{E}}_t \delta_{t+1} - \delta_t)]. \quad (20)$$

The log linearized Phillips curve is derived from firms' problem:

$$\hat{\pi}_t = \frac{\beta}{1 + \omega\beta} \tilde{\mathbb{E}}_t \hat{\pi}_{t+1} + \frac{\omega}{1 + \omega\beta} \hat{\pi}_{t-1} + \kappa \hat{y}_t, \quad (21)$$

Note that since the households own the firms, they share the same worst case belief. Equations (20) and (21) are similar to those derived in the the baseline model except that now the expectation is taken under the worst case belief.

Belief updating and solution of the model The fact that monetary signal is ambiguous complicates agents' belief updating. Given signals, agents choose a $\sigma_m \in [\bar{\sigma}_m, \underline{\sigma}_m]$ that leads to the worst belief. In the setup of this paper, the choice of σ_m depends on the sign monetary surprise. It is apparent that household's utility strictly increases in the unknown productivity trend g . Thus facing an ambiguous monetary signal, the natural initial guess of the worse case scenario for private agents is the σ_m leading to a lower posterior $g_t|t$. Hence the perceived volatility of monetary signal ($\tilde{\sigma}_m$) is kinked:

$$\tilde{\sigma}_m = \begin{cases} \bar{\sigma}_m, & \text{if monetary surprise} > 0 \\ \underline{\sigma}_m, & \text{if monetary surprise} < 0 \end{cases}. \quad (22)$$

Intuitively a negative (positive) surprise is partially perceived as bad (good) news shock, thus the worst case, up to the first order, is associated with a smaller (bigger) σ_m leading to larger (smaller) drop (rise) in expected trend.¹⁷

The equation (22) results into a a kinked belief updating equation:

$$\tilde{\mathbb{E}}_t(\cdot) = \begin{cases} \mathbb{E}(\cdot | \Omega_t, \bar{\sigma}_m), & \text{if monetary surprise} > 0 \\ \mathbb{E}(\cdot | \Omega_t, \underline{\sigma}_m), & \text{if monetary surprise} < 0 \end{cases}. \quad (23)$$

¹⁶Ilut and Schneider (2014) log linearize around the worst case steady state. This is not the case here because the fact signals are uncertain does not distort the steady state: in steady state information is perfect.

¹⁷I keep the analysis at the first order approximation because solving the model beyond the first order would require a nonlinear filter for the belief updating equation. Solving such model in general is still an open question.

With ambiguous monetary signal, the dynamics of the model change depending on the type and the sign of shocks. In response to perfectly observed shocks: ε_t^δ and ε_t^π the dynamic of the model is characterized by equation (11) and equation (12) i.e. the same as in the baseline model. In response to unobserved shocks: ε_t^g , ε_t^a and ε_t^m the dynamic of the model depends on the signs of the shocks. In the following, I discuss the dynamics of the economy in response to monetary shocks. The results can be easily generalized to any shocks.

Case 1 ($\varepsilon_t^m > 0$): a positive monetary shock (contractionary) results into positive monetary surprise at time t . This is partly perceived as good news shock. Thus agents revise their beliefs about g_t upwards. But since they are max-minimizer, they choose to distrust the monetary signal and update their beliefs using the prior of monetary signal under worst case scenario i.e. $\bar{\sigma}_m$. As a result, agents have made a mistake and have become over-optimistic. In the next period and all the periods after, over-optimistic agents will be "shocked" by negative monetary surprises and slowly realize that there was no shock to g at time t . This correction process from $t + 1$ on that is accompanied by negative monetary surprises is linked to a worst case scenario prior $\underline{\sigma}_m$. In sum, the solution of the model in this case is summarized in Proposition (1).

Proposition 1. *if $\varepsilon_t^m > 0$ the solution of the model follows:*

$$X_{t+j} = \begin{cases} \bar{A}X_{t+j-1} + \bar{B}U_{t+j}, & \text{for } j = 0 \\ \underline{A}X_{t+j-1} + \underline{B}U_{t+j}, & \text{for } j > 0 \end{cases}, \quad (24)$$

$$X_{t+j}^f = \begin{cases} \bar{F}X_{t+j}, & \text{for } j = 0 \\ \underline{F}X_{t+j}, & \text{for } j > 0 \end{cases}. \quad (25)$$

where \bar{A} , \bar{B} and \bar{F} are parameters associated with $\bar{\sigma}_m$ and \underline{A} , \underline{B} and \underline{F} are parameters associated with $\underline{\sigma}_m$. The agent chooses to commit on her choices of $\sigma_{m,t}$ made in the past.

Proof. see Appendix (B). ■

Case 2 ($\varepsilon_t^m < 0$): similarly to case 1, a negative monetary shock results into a negative monetary surprise at time t and positive surprise afterwards. The solution of the model in this case is summarized in Proposition (2).

Proposition 2. *if $(\varepsilon_t^a < 0$ or $\varepsilon_t^m < 0$) the solution of the model follows:*

$$X_{t+j} = \begin{cases} \underline{A}X_{t+j-1} + \underline{B}U_{t+j}, & \text{for } j = 0 \\ \bar{A}X_{t+j-1} + \bar{B}U_{t+j}, & \text{for } j > 0 \end{cases}, \quad (26)$$

$$X_{t+j}^f = \begin{cases} \underline{F}X_{t+j}, & \text{for } j = 0 \\ \bar{F}X_{t+j}, & \text{for } j > 0 \end{cases}. \quad (27)$$

8 Estimation and Results

Estimation Procedure I set majority of the parameters to be the median estimates of the baseline model, except for $\underline{\sigma}_m$ and $\bar{\sigma}_m$, which were not present in the linear model. I estimate those key parameters by minimizing the distance between the impact effects of monetary policy shock on real GDP forecast revisions generated from simulations of the model and those from actual data, i.e. those reported in Figure (4). As it is discussed in Proposition (1) and Proposition (2), the response to a positive monetary shock (the right panel in Figure (4)) is used to estimate $\bar{\sigma}_m$ and the left panel in Figure (4) is useful for the estimation of $\underline{\sigma}_m$. The key parameters to be estimated are stacked in vector $\Theta \equiv [\underline{\sigma}_m \quad \bar{\sigma}_m]$.

Let \mathbf{M}^* denote the six moments from data: three forecast revision horizons and two sets of estimates in response to positive and negative shocks. For a given parameter vector Θ , I simulate those moments $\mathbf{M}(\Theta)$ using the model. For each parameter in Θ , the distance between the actual data and the simulation of the model is:

$$\left(\frac{\mathbf{M}^*(i) - \mathbf{M}(\Theta)(i)}{\mathbf{M}^*(i)} \right)^2.$$

My estimator is the solution to the following problem:

$$\hat{\Theta} = \underset{\Theta}{\operatorname{argmin}} \sum_{i=1}^6 \left(\frac{\mathbf{M}^*(i) - \mathbf{M}(\Theta)(i)}{\mathbf{M}^*(i)} \right)^2.$$

Parameter estimates are reported in Table (4), in which the first column reports the mean estimate (σ_m) that is taken from Table (1) and second and third columns report the lower bound ($\underline{\sigma}_m$) and upper bound ($\bar{\sigma}_m$) respectively. While the estimated $\underline{\sigma}_m$ is merely slightly smaller than the mean estimate, $\bar{\sigma}_m$ is double the size of σ_m . This is the case because empirically the response of real GDP forecast revisions to negative monetary shocks are similar to those estimated in linear model and those responses to positive monetary shocks are much smaller.

Table 4: Estimated Parameters

The Mean Estimate σ_m	The Lower Bound $\underline{\sigma}_m$	The Upper Bound $\bar{\sigma}_m$
1.2×10^{-3}	1.0×10^{-3}	2.0×10^{-3}

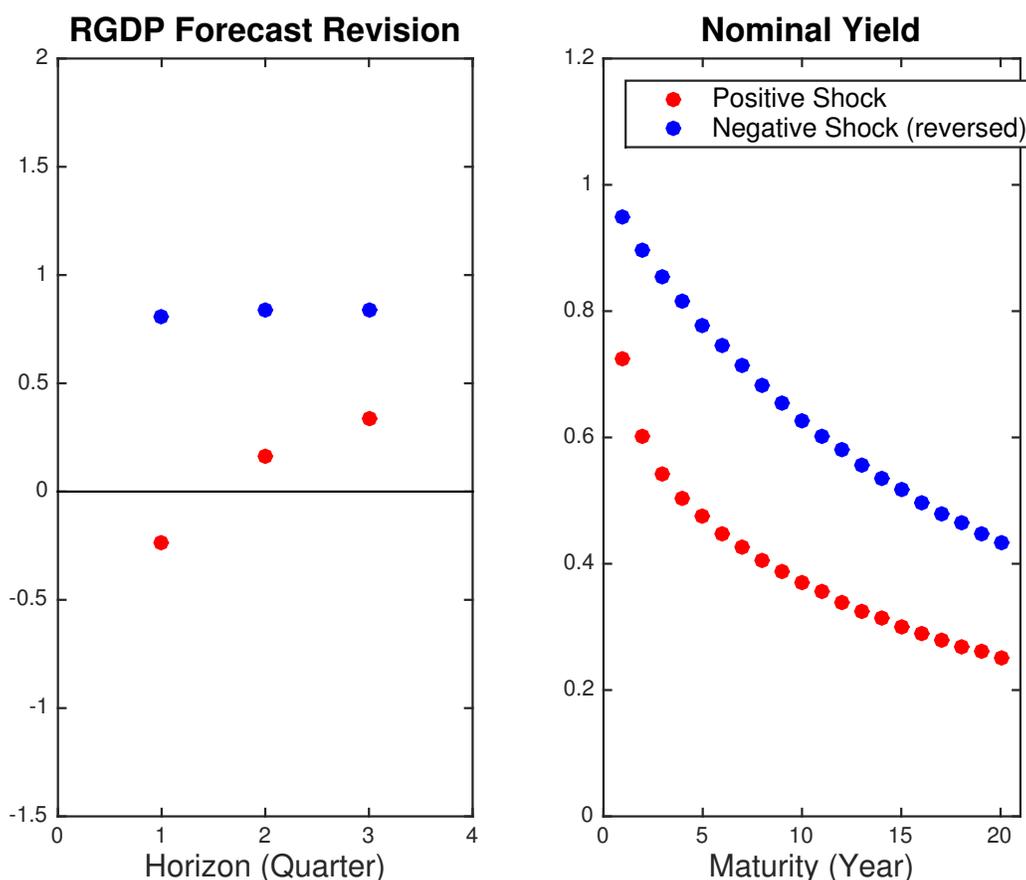
Notes: The mean estimate is taken from Table (1), the lower bound and upper bound reported in the second and third columns are estimated using the simulated method of moments.

Results: the Asymmetric Effects of Monetary Shocks The left panel in Figure (7) depicts the impact effects of monetary shocks on real GDP forecast revision at different horizons. And the the right panel reports the impact response of nominal yield with different maturities to monetary shocks. The red circles report those in response to a positive monetary shock and blue ones correspond to the responses to a negative monetary shock. The asymmetry is apparent and this figure successfully replicates the empirical fact that we have seen in Figure (4). The intuition is the following: a negative (positive) monetary shock is perceived by private agents as an ambiguous signal about bad(good) news shock therefore they become more pessimistic (optimistic), yet the correct amount of belief updating is ambiguous. Since they are ambiguity averse, they update as much (less) as possible, in other words the information channel is strong (weak). Therefore the puzzling facts are more (less) pronounced when the monetary shock is negative (positive). The right panel in Figure (7) confirms that the same asymmetric patterns hold for the yield curve: in absolute value, a negative monetary shock affects the yield curve more than the effects of a positive shock.

Figure (13) plots the impulse response functions of output and inflation to a positive (in red) and a negative monetary shock (in blue). As one can see, contractionary monetary policy shock is more effective (disturbing) than a expansionary one. This is consistent with the empirical studies for example [Barnichon and Matthes \(2016\)](#). This is a result of information channel and ambiguity aversion. Information channel offsets the transitional channel of monetary policy, together with the assumptions of ambiguous signal and ambiguity averse agent, the information channel is stronger when the monetary shock sends a bad news (negative monetary shock). Since the traditional channel of monetary shock is sign independent, overall the total effect is asymmetric.

Robustness Checks The results discussed above is based on a model in which monetary policy as a signal is ambiguous. However, this is not a necessary condition. The key of the success of the model is the kinked belief updating equation, which arises as long as one or many of the following

Figure 7: Model Result I: Rationalizing Asymmetric Facts



Note: The model implied responses of real GDP forecast revisions at different horizons and nominal yield with different maturities to monetary shocks.

volatilities are uncertain: level productivity shock, trend productivity shock, private signal shock and monetary shock. Figure (14) plots the estimation results of a model in which the volatility of the level productivity shock is uncertain, Figure (15) plots the results in the case where volatility of the trend productivity shock is uncertain, and in Figure (16) both private signal shock and monetary shock feature uncertain volatilities. In all of those cases, the model generates asymmetric effects of monetary shocks on real GDP forecast revision and the yield curve in a way consistent with empirical findings.

An interesting feature arises in Figure (14) and (16): the impulse responses of real GDP and inflation to a negative monetary shock are hump shaped. The theoretical literature rationalizes the hump shape IRFs by introducing capital or habit formation to the basic NK model. This paper provides an alternative mechanism: the information channel. Through the information channel, a

negative monetary shock has contractionary effects while through the standard channel the impacts are contractionary. After a pure monetary shock, in the beginning, information channel is strong and it nearly offsets the standard channel. The total effect is small on impact. Over the time, agents learn the truth and the information channel diminishes faster than the conventional effect, therefore even though the realized interest rates are not as high as in the first period the total effects of monetary policy are stronger in the subsequent periods.

9 Conclusion

I have provided a micro-founded model based on the information channel of monetary policy to rationalize: first monetary policy shocks have an impact on interest rates at long horizons (10 years or more) and second a contractionary monetary policy has expansionary effect on agents' expectations. In the framework, central banks holds superior information about future economic conditions. Policy actions partially reveal that information to the public, and thus the model is capable of generating the aforementioned puzzles. With the presence of the information channel, the impacts of monetary shocks on output and inflation are reduced. I have also discussed the optimal central bank communication and optimal monetary simple rule: it is optimal to be fully opaque about the trend shocks and the information asymmetry studied in this paper does not affect the design of optimal simple rule. Moreover, the model is capable of generating hump shape impulse responses without introducing habit formation or capital adjustment.

I have provided novel facts on the asymmetric impacts of monetary shocks on the yield curve and real GDP forecast revisions. In light of this asymmetry, I extend the model by introducing ambiguous signals and ambiguity averse agents. With those new features, the extended model rationalizes the novel puzzling asymmetric facts. In addition, it predicts the asymmetric effects of monetary shocks on output and inflation are asymmetric as it is shown in the literature.

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A Table

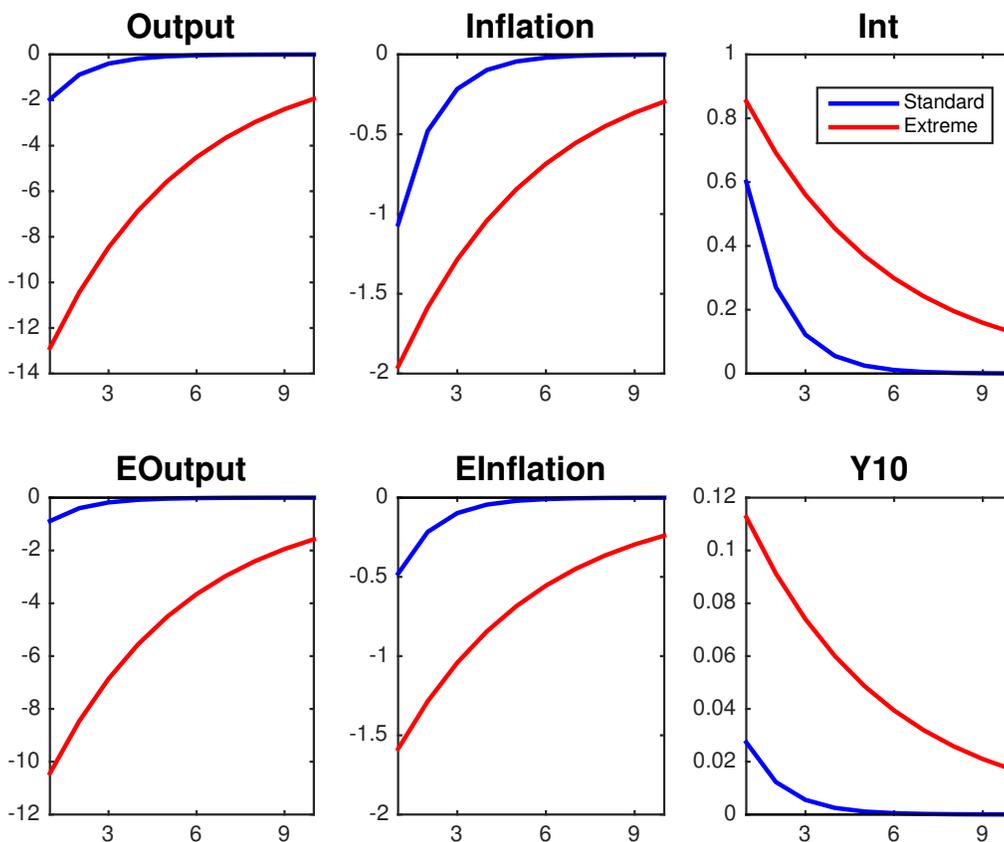
Table 5: Prior and Posterior: others

	Priors			Asymmetric Information		Perfect Information	
	Mean	s.d	Distribution	Mean	s.d	Mean	s.d
σ_π	2×10^{-3}	4	InvGamma	2.5×10^{-3}	0.2×10^{-3}	3.0×10^{-3}	0.3×10^{-3}
σ_d	2×10^{-3}	4	InvGamma	1.2×10^{-2}	0.3×10^{-2}	3.7×10^{-2}	0.4×10^{-2}
σ_{y2}^e	2×10^{-3}	4	InvGamma	5.5×10^{-5}	4.1×10^{-5}	6.4×10^{-5}	6.3×10^{-5}
σ_{y5}^e	2×10^{-3}	4	InvGamma	4.3×10^{-5}	3.2×10^{-5}	5.0×10^{-5}	4.3×10^{-5}
σ_{y10}^e	2×10^{-3}	4	InvGamma	6.8×10^{-4}	4.8×10^{-5}	2.0×10^{-4}	4.7×10^{-5}
g^*	0.5×10^{-2}	0.5×10^{-2}	Gamma	0.4×10^{-2}	0.7×10^{-2}	0.4×10^{-2}	0.6×10^{-2}
π^*	0.5×10^{-2}	0.5×10^{-2}	Gamma	0.6×10^{-2}	0.4×10^{-2}	0.6×10^{-2}	0.3×10^{-2}
β	0.99	0.01	Beta	0.99	0.01	0.99	0.01
k	0.3	0.1	Beta	0.12	0.04	0.36	0.07
ω	0.3	0.1	Beta	0.34	0.08	0.30	0.07
ρ_g	0.5	0.2	Beta	0.98	0.003	0.99	0.003
ρ_m	0.5	0.2	Beta	0.78	0.02	0.81	0.03
ρ_d	0.5	0.2	Beta	0.89	0.02	0.94	0.01
ϕ_π	1.5	0.25	Gamma	1.1	0.05	1.6	0.13
ϕ_y	0.25	0.05	Gamma	0.35	0.06	0.31	0.06
ϕ_r	1	0.05	Gamma	0.90	0.02	0.97	0.03

Note: Posterior means and standard deviations are estimated by Random Walk Metropolis-Hasting algorithm

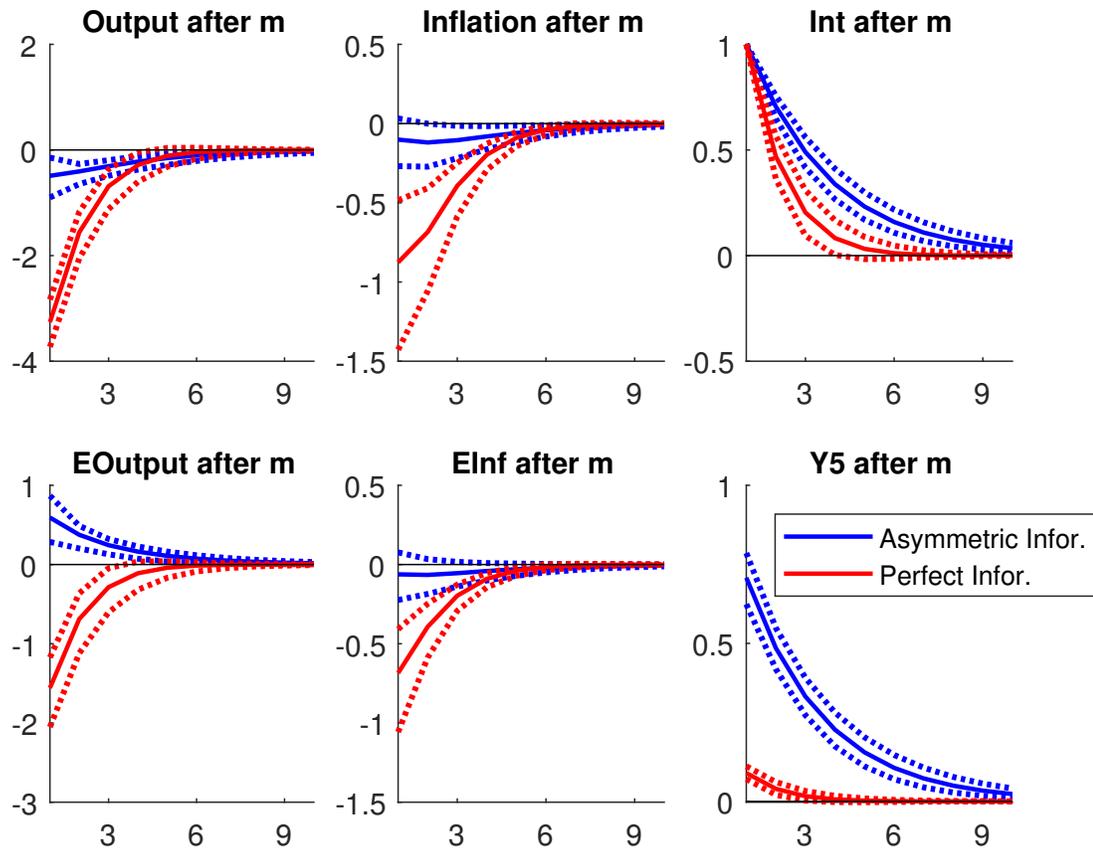
B Figures

Figure 8: Predictions of a Standard NK Model: Responses to Positive Monetary Shocks



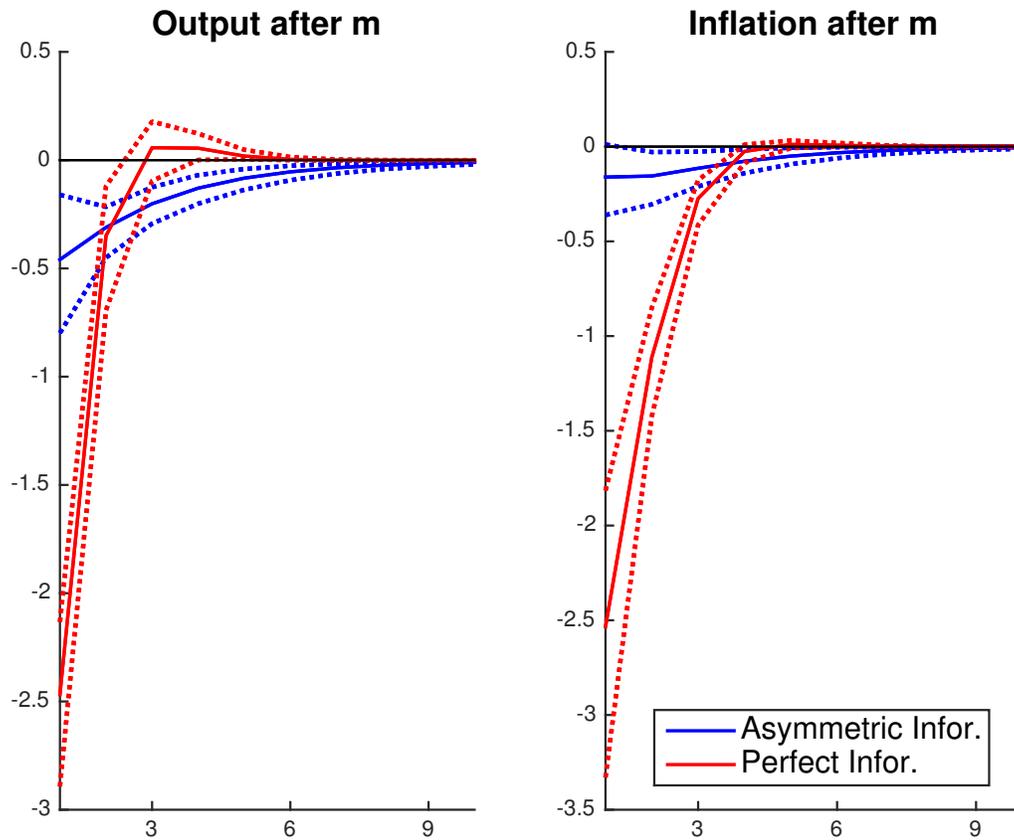
Note: This figure plots the IRFs to a positive monetary shock based on two sets of calibrations. The blue lines plot those based on a standard calibration, and the red lines plot those based on an extreme calibration namely $\kappa = 0.01$ and $\rho_m = 0.95$.

Figure 9: IRFs: Asymmetric Information v.s Perfect Information



Note: m , Int , $EOutput$, $EInf$ and $Y5$ denotes monetary shock denotes monetary shock, the policy rate, expected log real output at one quarter ahead, expected inflation at one quarter ahead and five years nominal yield respectively. The solid and dashed lines are the mean estimates and 95% confidence interval of IRFs of the models constructed based on the posterior parameters' distributions. The ones in blue plot the predictions of the model under asymmetric information. The red ones plot those for the perfect information case: all parameters are set to be the same as for the asymmetric case except for the volatility for private signal σ^p , which is imposed to be zero.

Figure 10: Asymmetric Information v.s Perfect Information: Separate Estimations



This figure plots the IRFs for both the model with asymmetric information (in blue) and perfect information (in red). Both models are estimated separately. The solid and dashed lines are the mean estimates and 95% confidence interval of IRFs of the models constructed based on the posterior parameters' distributions.

Figure 11: Optimal Central Bank Communication

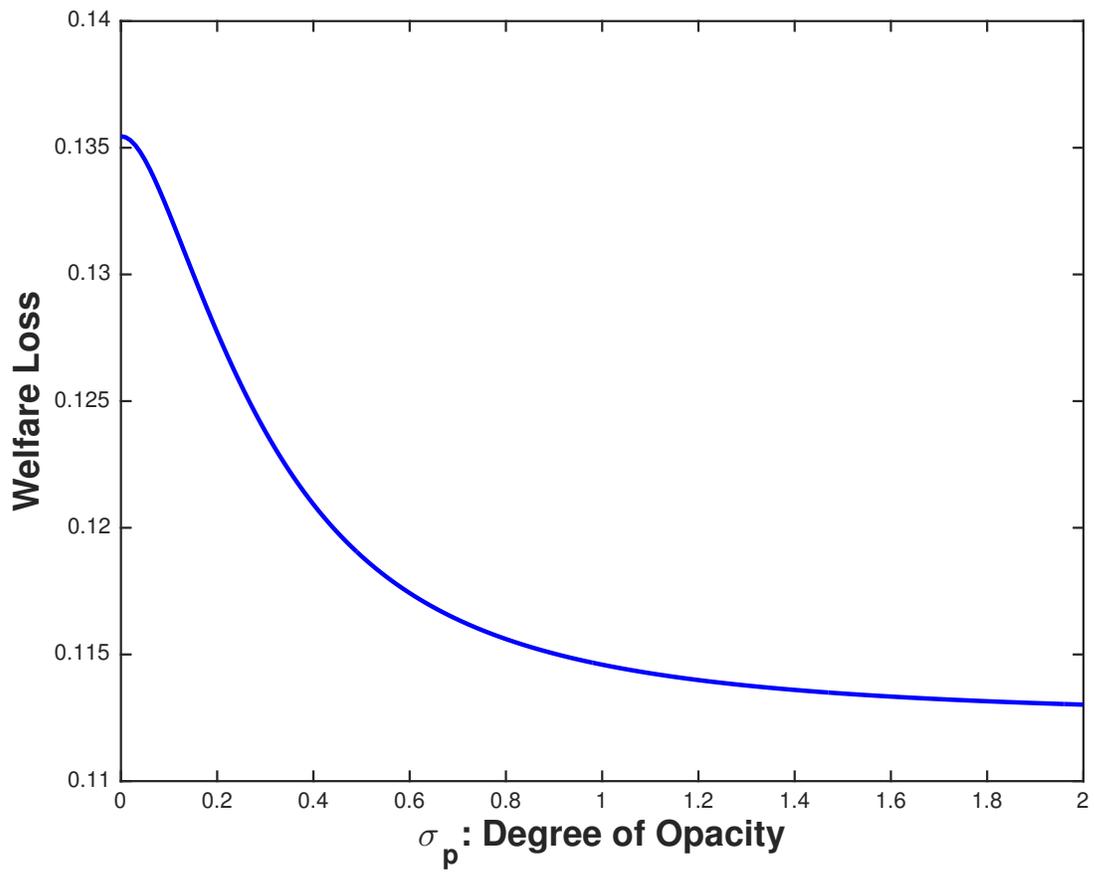


Figure 12: Optimal Monetary Policy Simple Rule

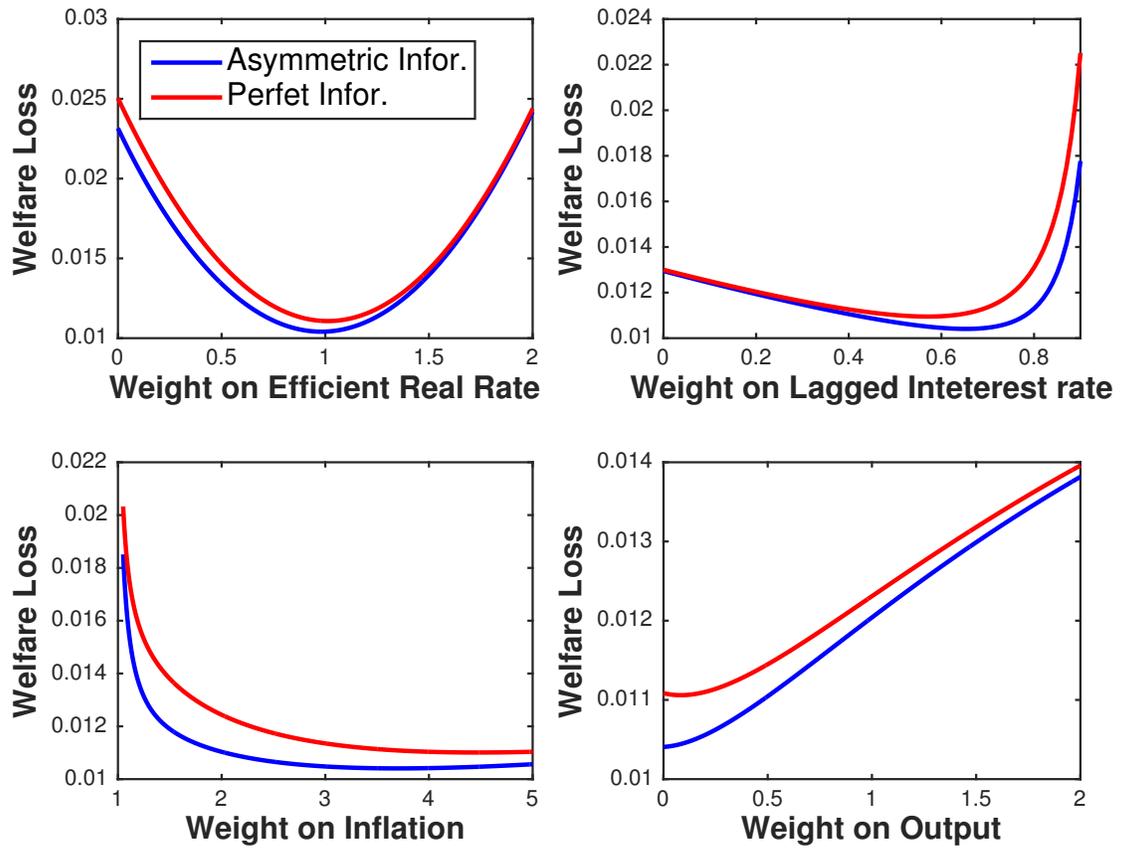


Figure 13: Model Result II: the Asymmetric Effect on Economic Activities

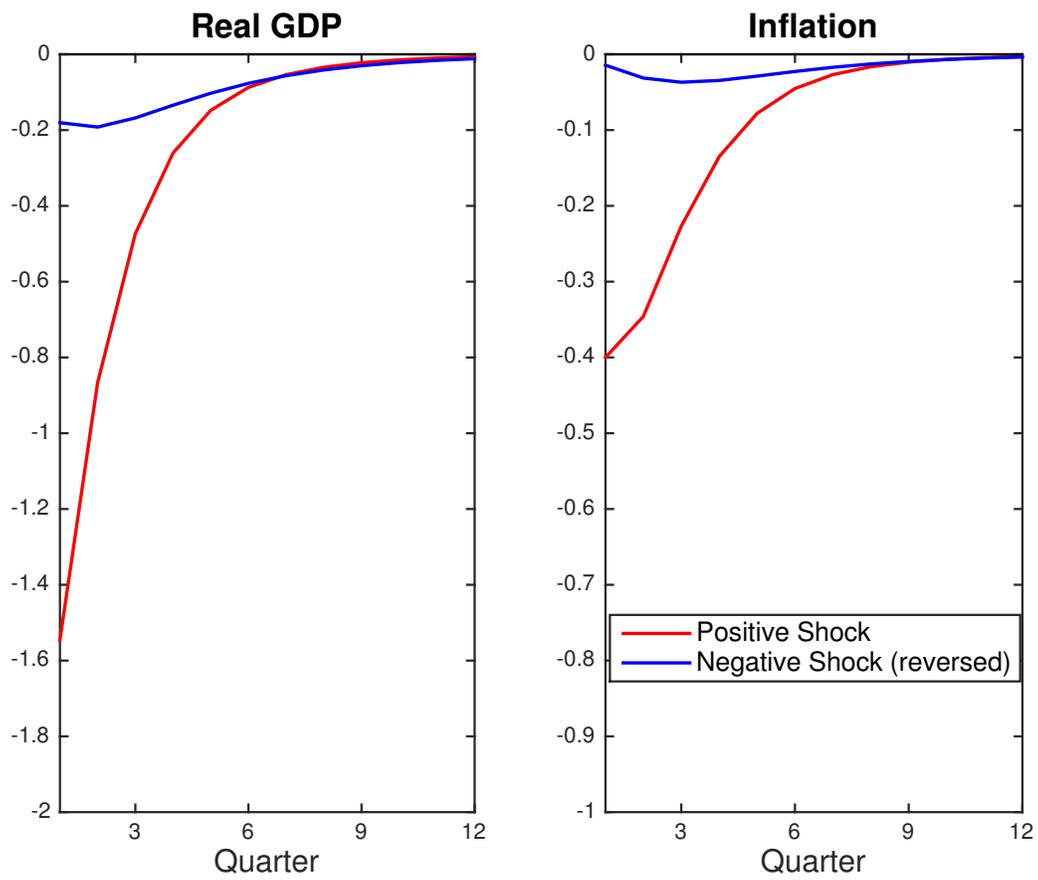


Figure 14: Model Robustness Check I: Ambiguous Productivity Volatility

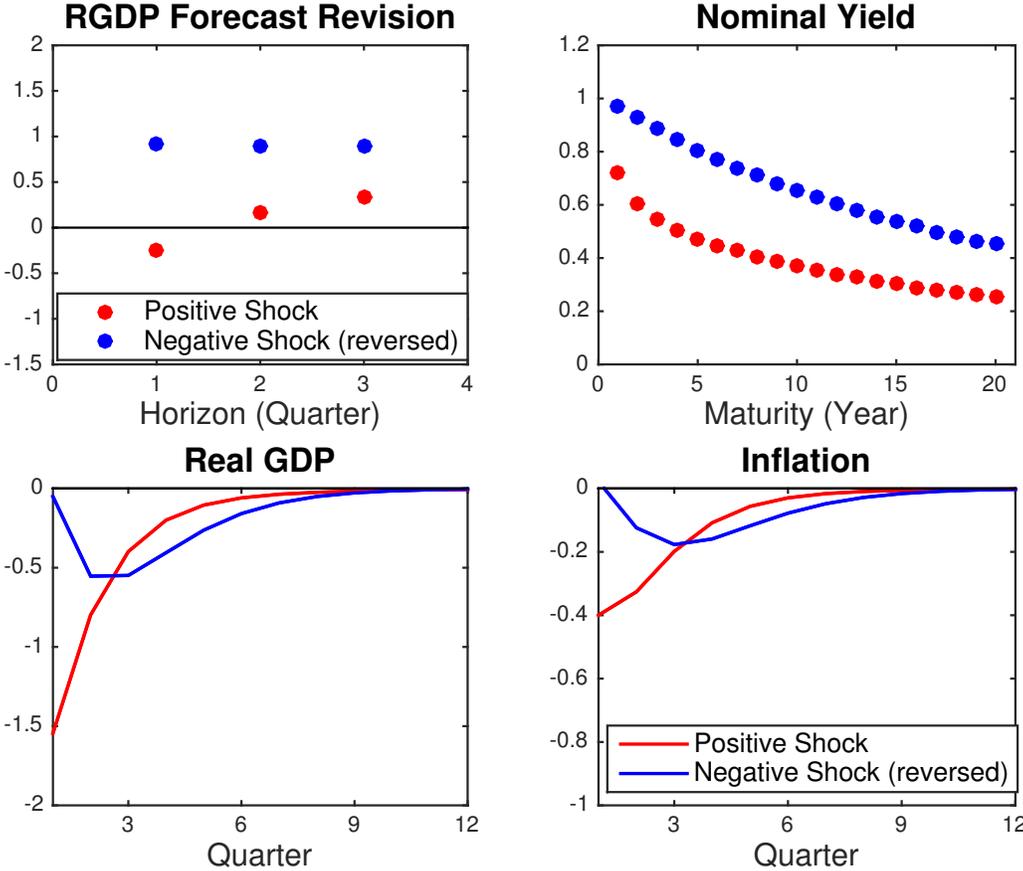


Figure 15: Model Robustness Check II: Ambiguous Growth Volatility

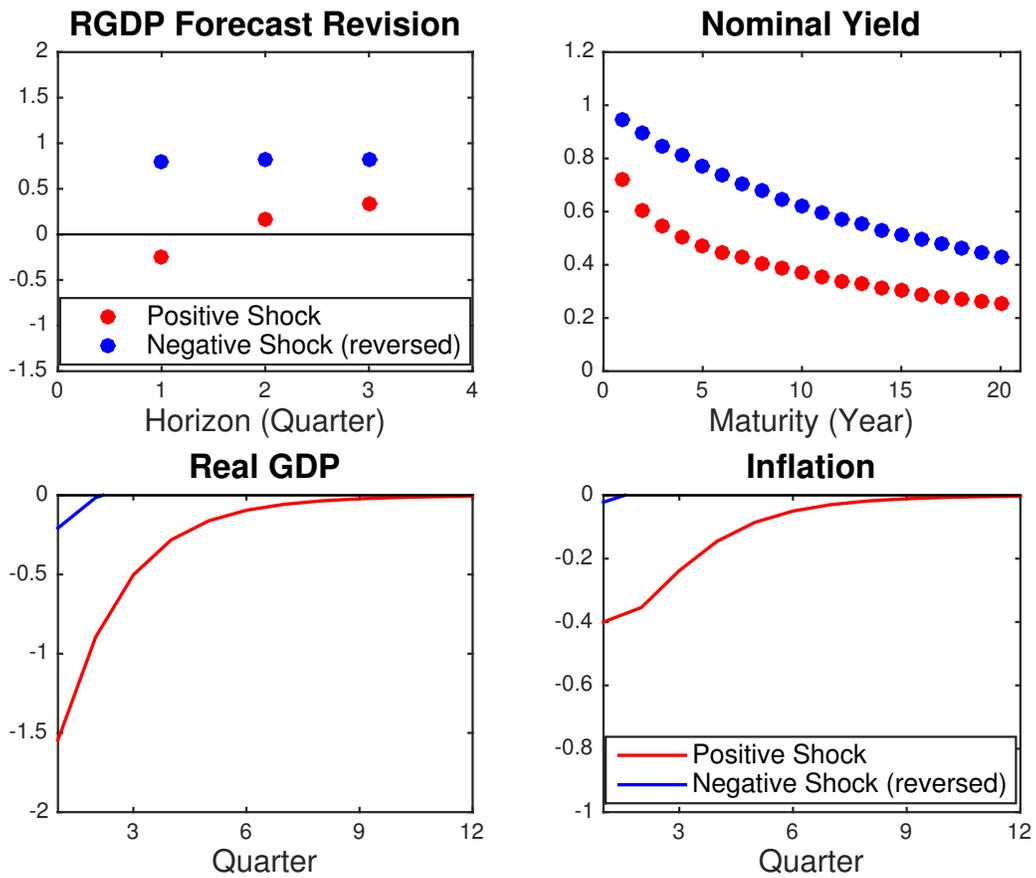


Figure 16: Model Robustness Check III: Ambiguous Monetary and Private Signal Volatilities

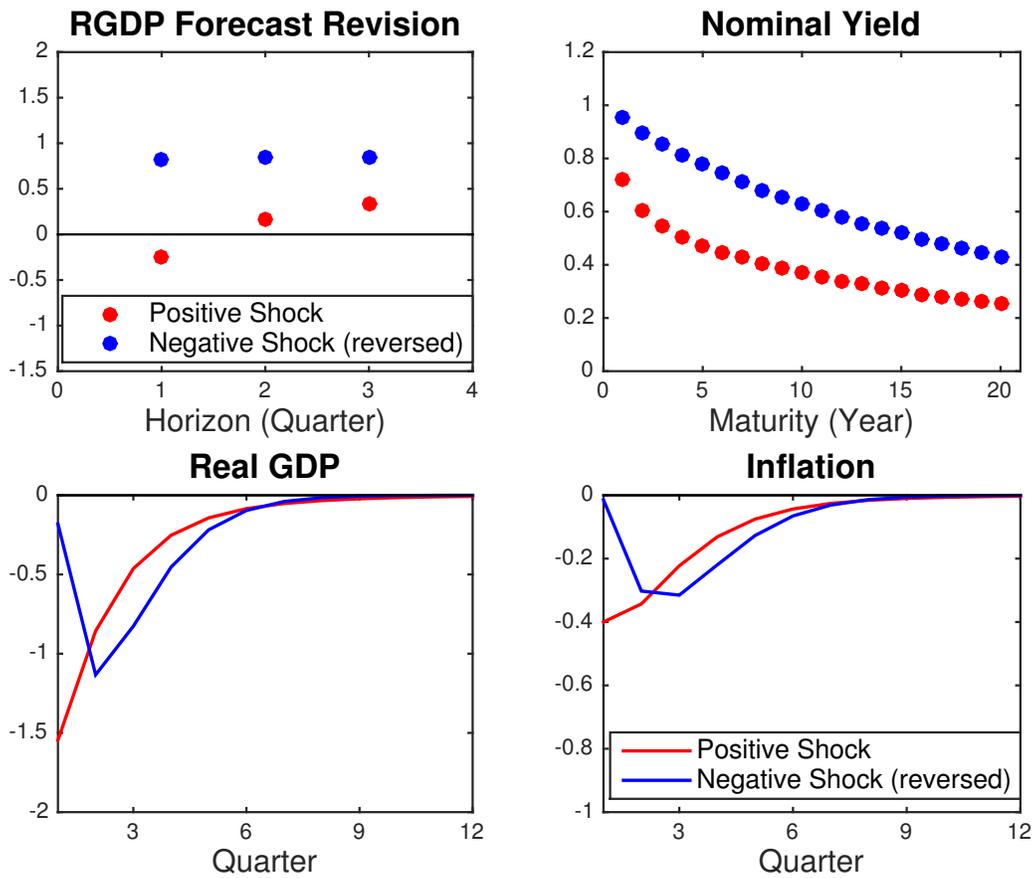
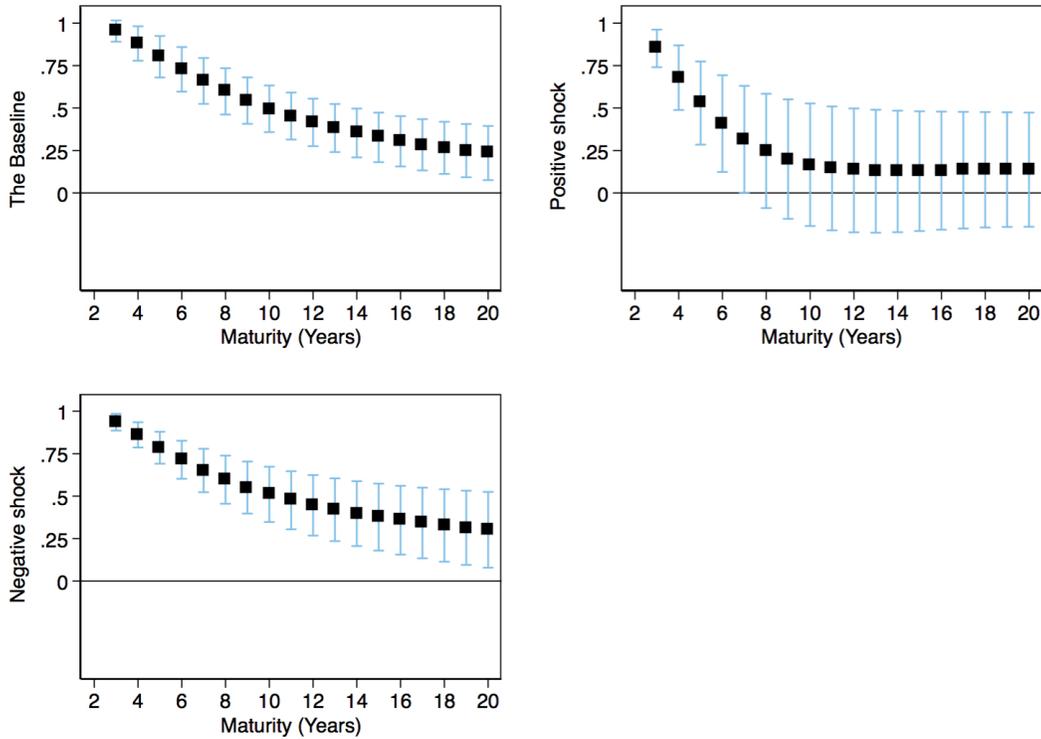
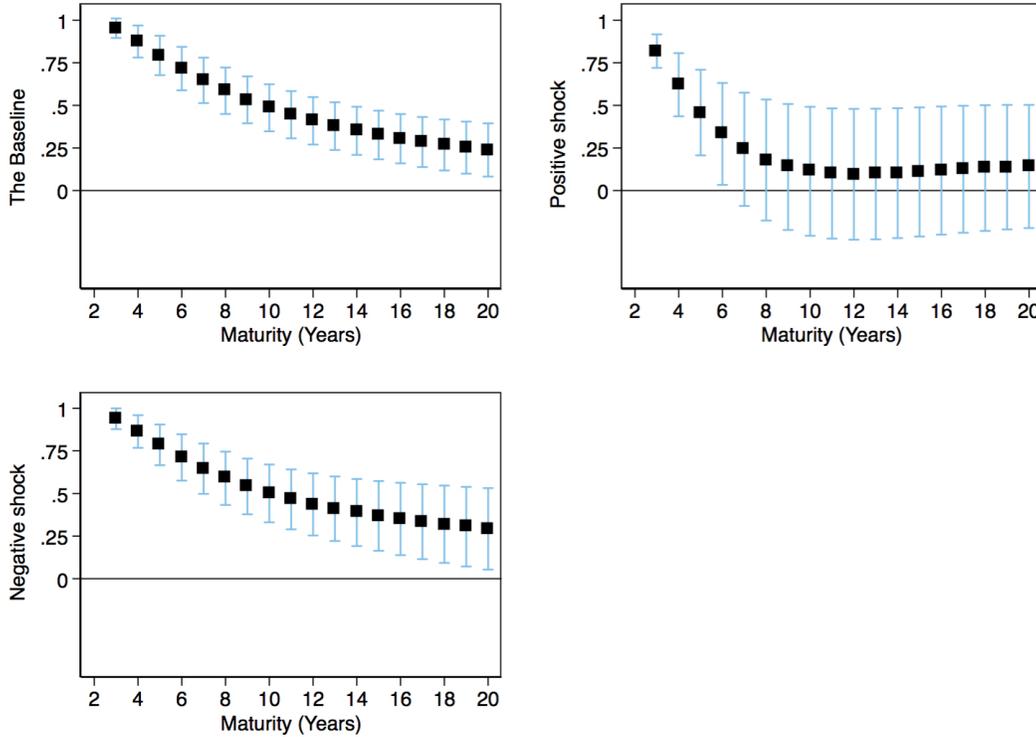


Figure 17: Impact Effect on Yield Robustness Check I: Excluding Recession Periods



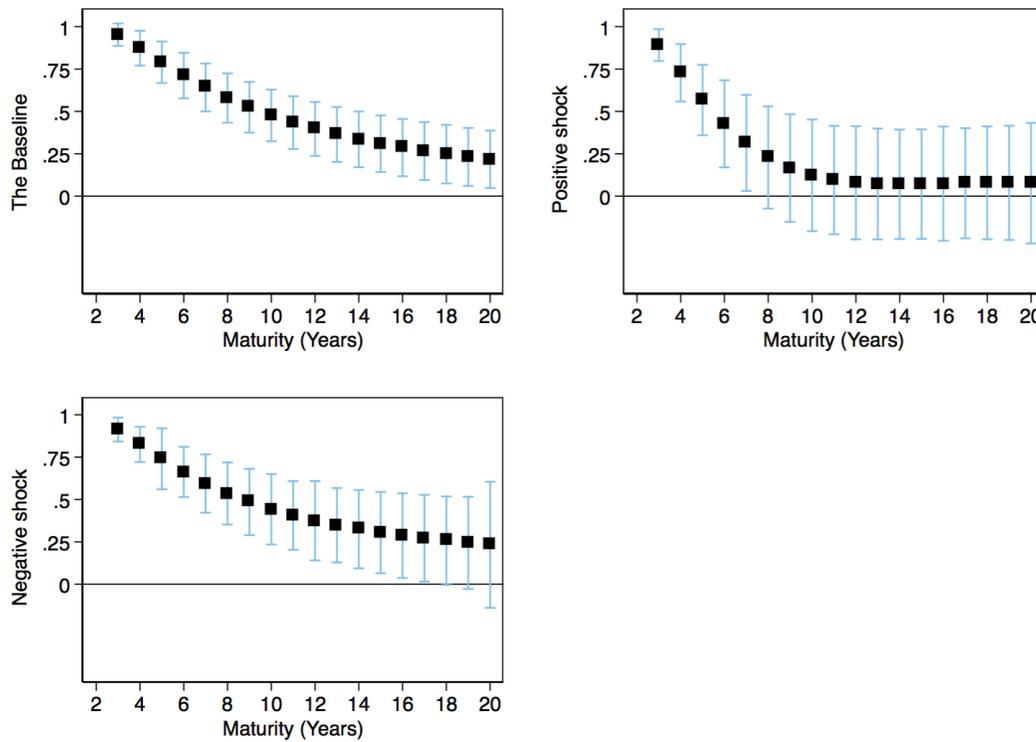
Notes: The top left panel reports the results from estimating the baseline regressions: $\Delta Y_t^h = \alpha + \beta_h \Delta MP_t + Factors + \varepsilon_{h,t}$ using HFI monetary surprises as instruments for ΔMP_t . The square dots on the bottom panel represent the estimated $\hat{\beta}_1^h + \hat{\beta}_2^h$ and the square dots on the top right panel represent the estimated $\hat{\beta}_1^h$, where $\hat{\beta}^h$ s are estimated from separated regressions: $\Delta Y_t^h = \alpha_1^h + \alpha_2^h I_{negative} + \beta_1^h \Delta MP_t + \beta_2^h \Delta MP_t \times I_{negative} + Factors + v_{h,t}$ using HFI monetary surprises and those interacted $I_{negative}$ with as instruments for the variables of interest ΔMP_t and $\Delta MP_t \times I_{negative}$. The blue lines are the 95% confidence interval constructed using standard errors that are robust to serial correlation and heteroskedasticity. Sample: 1995M1 to 2015M12.

Figure 18: Impact Effect on Yield Robustness Check II: Control for Eight Factors



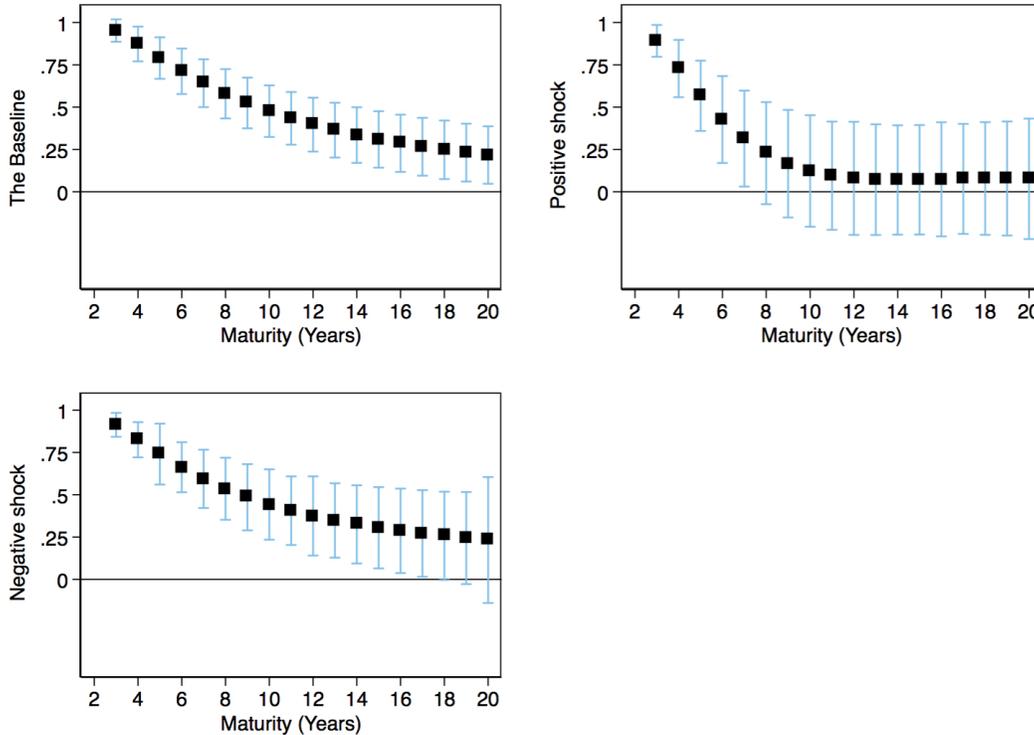
Note: The top left panel reports the results from estimating the baseline regressions: $\Delta Y_t^h = \alpha + \beta_h \Delta MP_t + Factors + \varepsilon_{h,t}$ using HFI monetary surprises as instruments for ΔMP_t . The control variable *Factors* contains eight factors. The square dots on the bottom panel represent the estimated $\hat{\beta}_1^h + \hat{\beta}_2^h$ and the square dots on the top right panel represent the estimated $\hat{\beta}_1^h$, where $\hat{\beta}^h$ s are estimated from separated regressions: $\Delta Y_t^h = \alpha_1^h + \alpha_2^h I_{negative} + \beta_1^h \Delta MP_t + \beta_2^h \Delta MP_t \times I_{negative} + Factors + v_{h,t}$ using HFI monetary surprises and those interacted $I_{negative}$ with as instruments for the variables of interest ΔMP_t and $\Delta MP_t \times I_{negative}$. The blue lines are the 95% confidence interval constructed using standard errors that are robust to serial correlation and heteroskedasticity. Sample: 1995M1 to 2015M12.

Figure 19: Impact Effect on Yield Robustness Check III: Control for Greenbook Forecasts



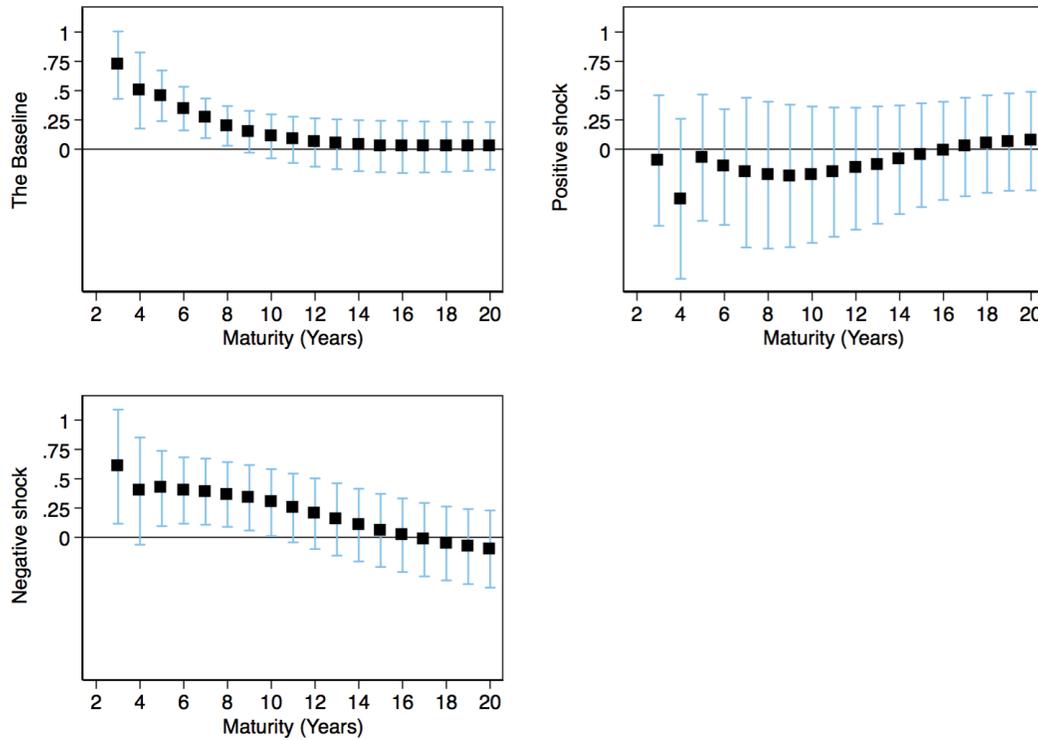
Note: The top left panel reports the results from estimating the baseline regressions: $\Delta Y_t^h = \alpha + \beta_h \Delta MP_t + Factors + GB + \varepsilon_{h,t}$ using HFI monetary surprises as instruments for ΔMP_t . The Federal Reserve Bank’s staff forecasts (Greenbook forecasts), namely the forecasts of real GDP, unemployment and inflation, are included as controls. The square dots on the bottom panel represent the estimated $\hat{\beta}_1^h + \hat{\beta}_2^h$ and the square dots on the top right panel represent the estimated $\hat{\beta}_1^h$, where $\hat{\beta}^h$ s are estimated from separated regressions: $\Delta Y_t^h = \alpha_1^h + \alpha_2^h I_{negative} + \beta_1^h \Delta MP_t + \beta_2^h \Delta MP_t \times I_{negative} + Factors + GB + v_{h,t}$ using HFI monetary surprises and those interacted $I_{negative}$ with as instruments for the variables of interest ΔMP_t and $\Delta MP_t \times I_{negative}$. The blue lines are the 95% confidence interval constructed using standard errors that are robust to serial correlation and heteroskedasticity. Sample: 1995M1 to 2011M12.

Figure 20: Impact Effect on Yield Robustness Check IV: Excluding Control Variables



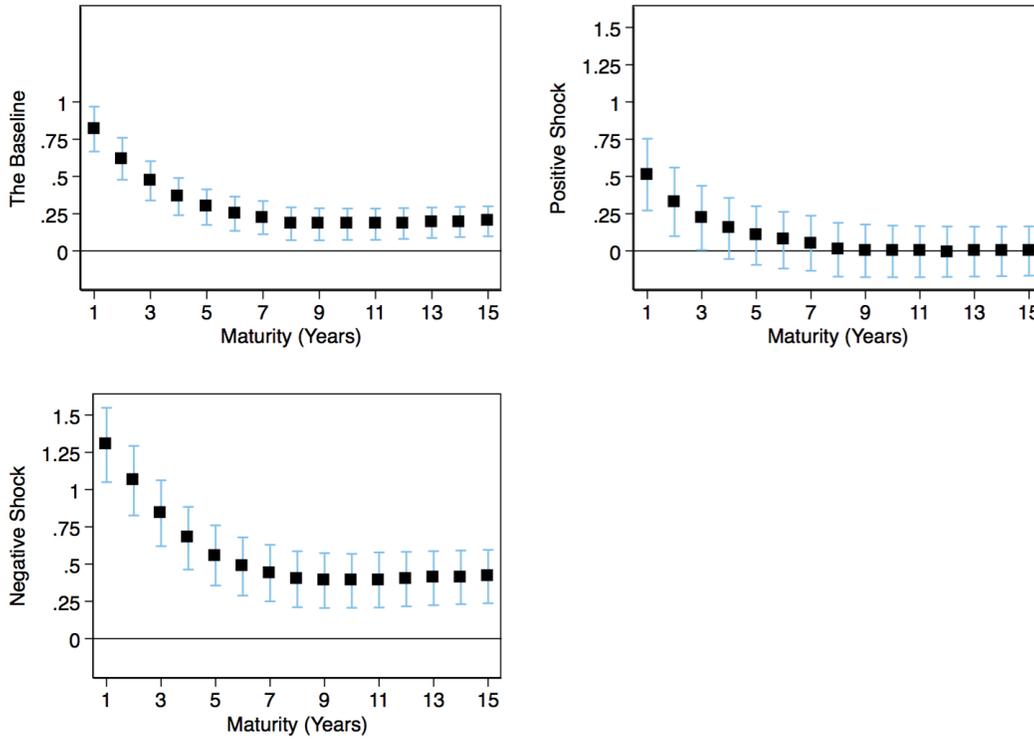
Note: The top left panel reports the results from estimating the baseline regressions: $\Delta Y_t^h = \alpha + \beta_h \Delta MP_t + \varepsilon_{h,t}$ using HFI monetary surprises as instruments for ΔMP_t . The square dots on the bottom panel represent the estimated $\hat{\beta}_1^h + \hat{\beta}_2^h$ and the square dots on the top right panel represent the estimated $\hat{\beta}_1^h$, where $\hat{\beta}^h$ s are estimated from separated regressions: $\Delta Y_t^h = \alpha_1^h + \alpha_2^h I_{negative} + \beta_1^h \Delta MP_t + \beta_2^h \Delta MP_t \times I_{negative} + v_{h,t}$ using HFI monetary surprises and those interacted $I_{negative}$ with as instruments for the variables of interest ΔMP_t and $\Delta MP_t \times I_{negative}$. The blue lines are the 95% confidence interval constructed using standard errors that are robust to serial correlation and heteroskedasticity. Sample: 1995M1 to 2015M12.

Figure 21: Impact Effect on Yield Robustness Check V: TIPSF



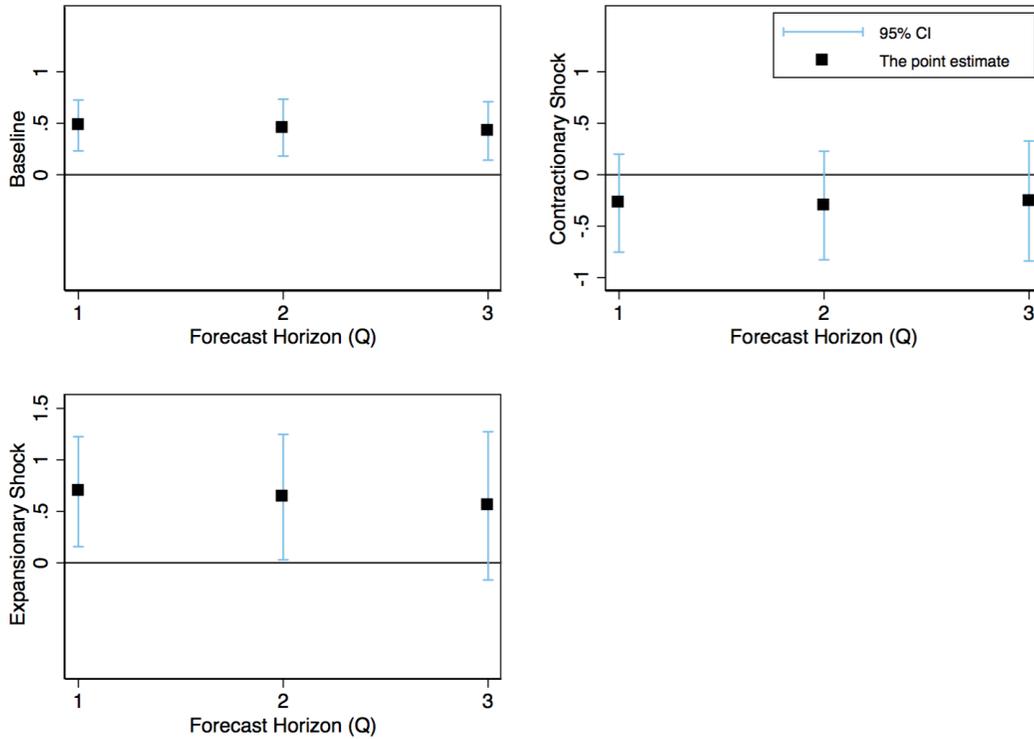
Note: The top left panel reports the results from estimating the baseline regressions: $\Delta Y_t^h = \alpha + \beta_h \Delta MP_t + Factors + \varepsilon_{h,t}$ using HFI monetary surprises as instruments for ΔMP_t . ΔY_t^h is the daily change around FOMC event in the Forward Rate constructed from Inflation Index bond (TIPSF). The square dots on the bottom panel represent the estimated $\hat{\beta}_1^h + \hat{\beta}_2^h$ and the square dots on the top right panel represent the estimated $\hat{\beta}_1^h$, where $\hat{\beta}^h$ s are estimated from separated regressions: $\Delta Y_t^h = \alpha_1^h + \alpha_2^h I_{negative} + \beta_1^h \Delta MP_t + \beta_2^h \Delta MP_t \times I_{negative} + Factors + v_{h,t}$ using HFI monetary surprises and those interacted $I_{negative}$ with as instruments for the variables of interest ΔMP_t and $\Delta MP_t \times I_{negative}$. The blue lines are the 95% confidence interval constructed using standard errors that are robust to serial correlation and heteroskedasticity. Sample: 2004M1 to 2015M12.

Figure 22: Impact Effect on Yield Robustness Check VI: R&R Monetary Shocks



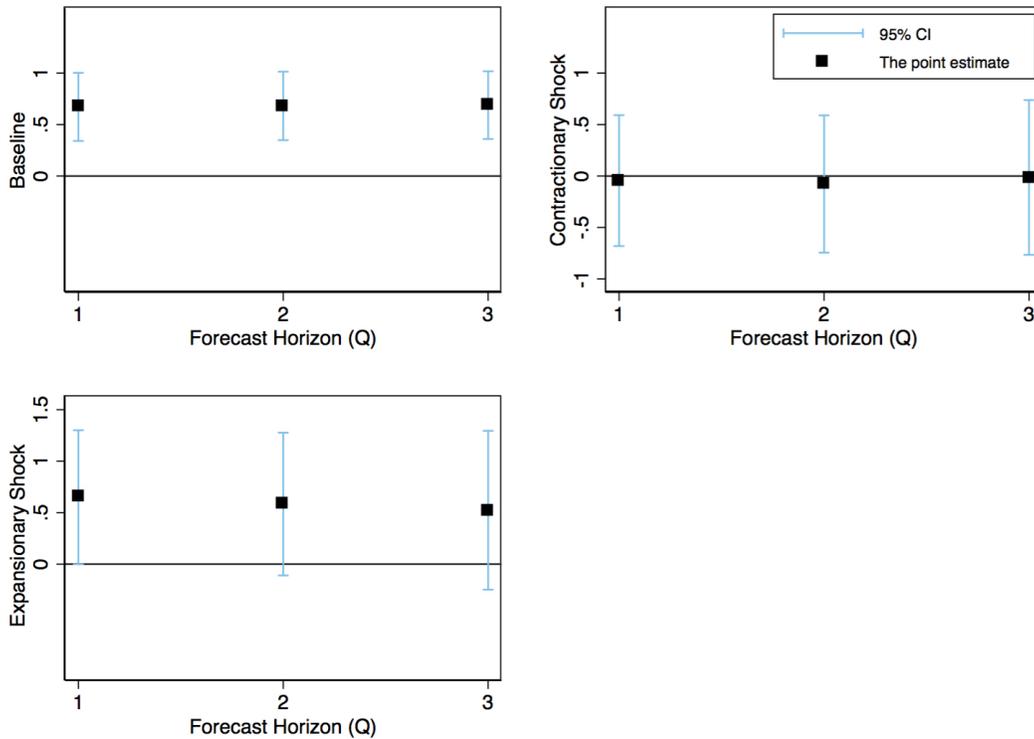
Note: The top left panel reports the results from estimating the baseline regressions: $\Delta Y_t^h = \alpha + \beta_h RR_t + Factors + \varepsilon_{h,t}$, where RR_t denotes the R&R Monetary Shocks. The square dots on the bottom panel represent the estimated $\hat{\beta}_1^h + \hat{\beta}_2^h$ and the square dots on the top right panel represent the estimated $\hat{\beta}_1^h$, where $\hat{\beta}^h$ s are estimated from separated regressions: $\Delta Y_t^h = \alpha_1^h + \alpha_2^h I_{negative} + \beta_1^h \Delta RR_t + \beta_2^h RR_t \times I_{negative} + v_{h,t}$. The blue lines are the 95% confidence interval constructed using standard errors that are robust to serial correlation and heteroskedasticity. Sample: 1969M1 to 2007M12.

Figure 23: Impact Effect on Forecast Revision Robustness Check I: Excluding Recession Periods



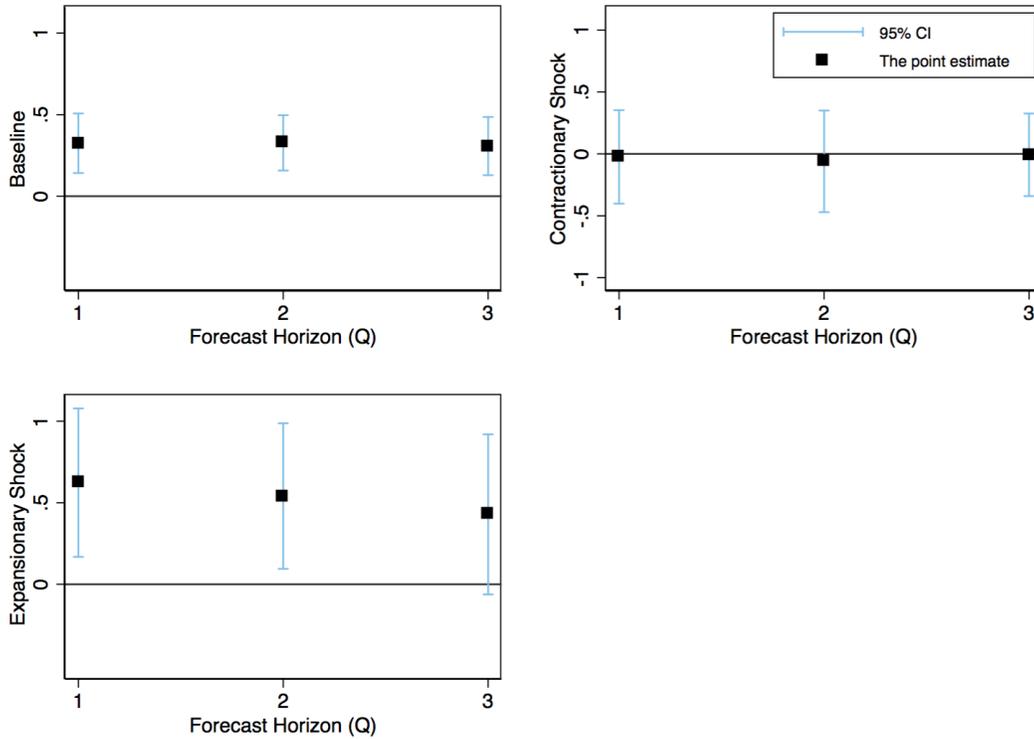
Note: The top left panel reports the results from estimating the baseline regressions: $y_{t+j|t} - y_{t+j|t-1} = \eta + \beta_j \Delta MP_t + controls_t + v_{h,t}$ using HFI monetary surprises as instruments for ΔMP_t . The square dots on the bottom panel represent the estimated $\hat{\beta}_1^h + \hat{\beta}_2^h$ and the square dots on the top right panel represent the estimated $\hat{\beta}_1^h$, where $\hat{\beta}^h$ s are estimated from separated regressions: $y_{t+j|t} - y_{t+j|t-1} = \alpha_1 + \beta_1^h \Delta MP_t + \beta_2^h \Delta MP_t \times I_{negative} + controls_t + v_{h,t}$ using HFI monetary surprises and those interacted $I_{negative}$ with as instruments for the variables of interest ΔMP_t and $\Delta MP_t \times I_{negative}$. The blue lines are the 95% confidence interval constructed using standard errors that are robust to serial correlation and heteroskedasticity. Sample: 1995Q1 to 2015Q4.

Figure 24: Impact Effect on Forecast Revision Robustness Check II: Control for Eight Factors



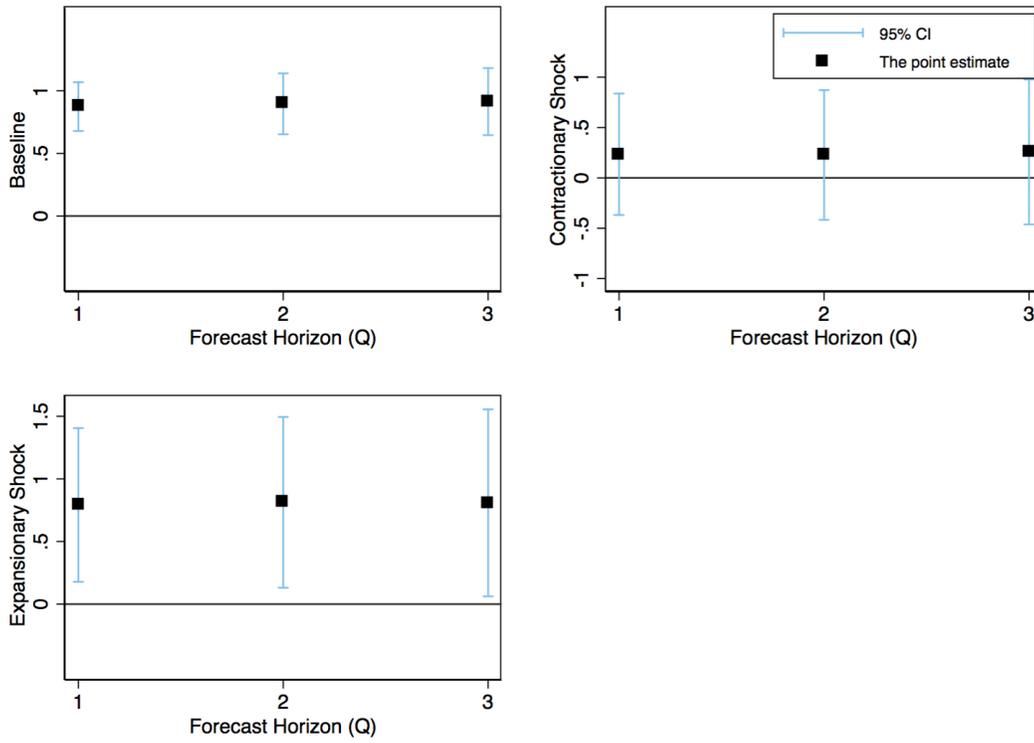
Note: The top left panel reports the results from estimating the baseline regressions: $y_{t+j|t} - y_{t+j|t-1} = \eta + \beta_j \Delta MP_t + controls_t + v_{h,t}$ using HFI monetary surprises as instruments for ΔMP_t . The square dots on the bottom panel represent the estimated $\hat{\beta}_1^h + \hat{\beta}_2^h$ and the square dots on the top right panel represent the estimated $\hat{\beta}_1^h$, where $\hat{\beta}^h$ s are estimated from separated regressions: $y_{t+j|t} - y_{t+j|t-1} = \alpha_1 + \beta_1^h \Delta MP_t + \beta_2^h \Delta MP_t \times I_{negative} + controls_t + v_{h,t}$ using HFI monetary surprises and those interacted $I_{negative}$ with as instruments for the variables of interest ΔMP_t and $\Delta MP_t \times I_{negative}$. The blue lines are the 95% confidence interval constructed using standard errors that are robust to serial correlation and heteroskedasticity. Sample: 1995Q1 to 2015Q4.

Figure 25: Impact Effect on Forecast Revision Robustness Check II: Control for Greenbook Forecasts



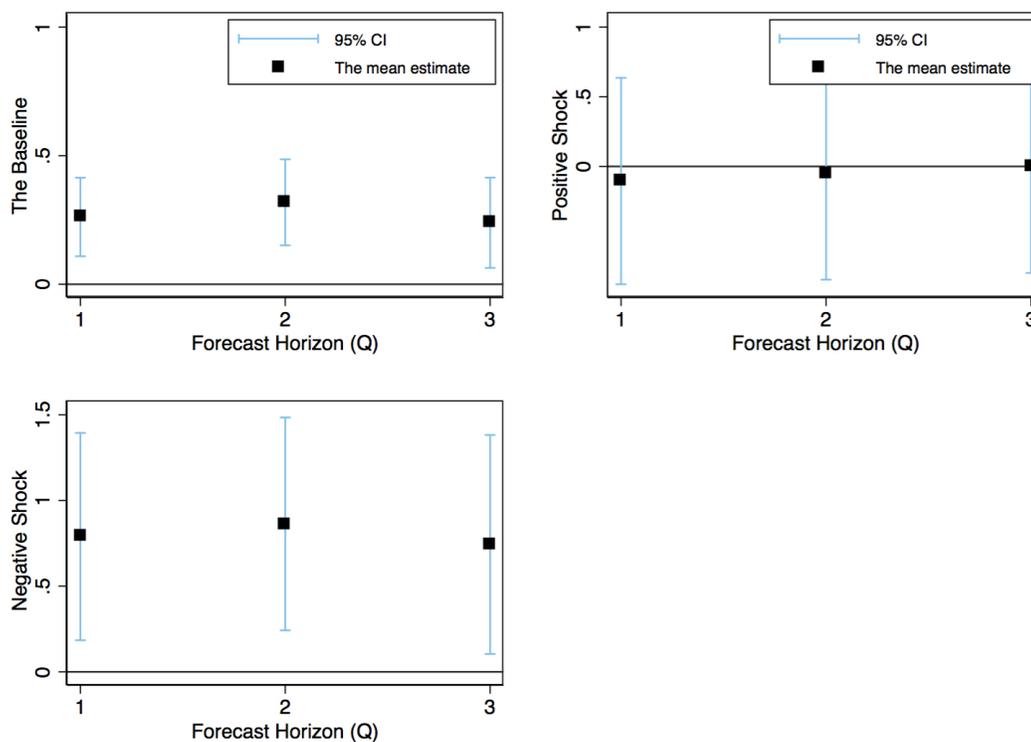
Note: The top left panel reports the results from estimating the baseline regressions: $y_{t+j|t} - y_{t+j|t-1} = \eta + \beta_j \Delta MP_t + controls_t + GB + v_{h,t}$ using HFI monetary surprises as instruments for ΔMP_t . The square dots on the bottom panel represent the estimated $\hat{\beta}_1^h + \hat{\beta}_2^h$ and the square dots on the top right panel represent the estimated $\hat{\beta}_1^h$, where $\hat{\beta}^h$ s are estimated from separated regressions: $y_{t+j|t} - y_{t+j|t-1} = \alpha_1 + \beta_1^h \Delta MP_t + \beta_2^h \Delta MP_t \times I_{negative} + controls_t + GB + v_{h,t}$ using HFI monetary surprises and those interacted $I_{negative}$ with as instruments for the variables of interest ΔMP_t and $\Delta MP_t \times I_{negative}$. The blue lines are the 95% confidence interval constructed using standard errors that are robust to serial correlation and heteroskedasticity. Sample: 1995Q1 to 2011Q4.

Figure 26: Impact Effect on Forecast Revision Robustness Check III: Excluding Control Variables



Note: The top left panel reports the results from estimating the baseline regressions: $y_{t+j|t} - y_{t+j|t-1} = \eta + \beta_j \Delta MP_t + v_{h,t}$ using HFI monetary surprises as instruments for ΔMP_t . The square dots on the bottom panel represent the estimated $\hat{\beta}_1^h + \hat{\beta}_2^h$ and the square dots on the top right panel represent the estimated $\hat{\beta}_1^h$, where $\hat{\beta}^h$ s are estimated from separated regressions: $y_{t+j|t} - y_{t+j|t-1} = \alpha_1 + \beta_1^h \Delta MP_t + \beta_2^h \Delta MP_t \times I_{negative} + v_{h,t}$ using HFI monetary surprises and those interacted $I_{negative}$ with as instruments for the variables of interest ΔMP_t and $\Delta MP_t \times I_{negative}$. The blue lines are the 95% confidence interval constructed using standard errors that are robust to serial correlation and heteroskedasticity. Sample: 1995Q1 to 2015Q4.

Figure 27: Impact Effect on Forecast Revision Robustness Check IV: R&R Monetary Shocks



Note: The top left panel reports the results from estimating the baseline regressions: $y_{t+j|t} - y_{t+j|t-1} = \eta + \beta_j RR_t + controls_t + v_{h,t}$ using HFI monetary surprises as instruments for ΔMP_t . The square dots on the bottom panel represent the estimated $\hat{\beta}_1^h + \hat{\beta}_2^h$ and the square dots on the top right panel represent the estimated $\hat{\beta}_1^h$, where $\hat{\beta}^h$ s are estimated from separated regressions: $y_{t+j|t} - y_{t+j|t-1} = \alpha_1 + \beta_1^h RR_t + \beta_2^h RR_t \times I_{negative} + controls_t + v_{h,t}$. The blue lines are the 95% confidence interval constructed using standard errors that are robust to serial correlation and heteroskedasticity. Sample: 1969Q1 to 2007Q4.

A Model Solution

Define: $X_t^b = (\varepsilon_t^m, \hat{g}_t, \delta_t, \varepsilon_t^\pi, \hat{i}_{t-1}, \hat{\pi}_{t-1})'$ and $X_t^f = (\hat{y}_t, \hat{\pi}_t)'$. The model can be summarized as:

$$M^0 \begin{bmatrix} X_{t+1}^b \\ X_{t+1|t}^f \end{bmatrix} = M^1 \begin{bmatrix} X_t^b \\ X_t^f \end{bmatrix} + M^2 \begin{bmatrix} X_{t|t}^b \\ X_t^f \end{bmatrix} + M^3 u_{t+1} \quad (1)$$

or explicitly:

$$M^0 \begin{bmatrix} \varepsilon_{t+1}^m \\ \hat{g}_{t+1} \\ \delta_{t+1} \\ \varepsilon_{t+1}^\pi \\ \hat{i}_t \\ \hat{\pi}_t \\ \hat{y}_{t+1|t} \\ \hat{\pi}_{t+1|t} \end{bmatrix} = M^1 \begin{bmatrix} \varepsilon_t^m \\ \hat{g}_t \\ \delta_t \\ \varepsilon_t^\pi \\ \hat{i}_{t-1} \\ \hat{\pi}_{t-1} \\ \hat{y}_t \\ \hat{\pi}_t \end{bmatrix} + M^2 \begin{bmatrix} \varepsilon_{t|t}^m \\ \hat{g}_{t|t} \\ \delta_t \\ \varepsilon_t^\pi \\ \hat{i}_{t-1} \\ \hat{\pi}_{t-1} \\ \hat{y}_t \\ \hat{\pi}_t \end{bmatrix} + M^3 \begin{bmatrix} \varepsilon_{t+1}^g \\ \varepsilon_{t+1}^\delta \\ \varepsilon_{t+1}^\pi \\ \varepsilon_{t+1}^m \end{bmatrix},$$

where

$$\begin{aligned}
M^0 &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & \frac{\beta}{1+\omega\beta} \end{bmatrix} & M_3 &= \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\
M^1 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_g & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_\delta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & (1-\rho_m)\rho_g & (1-\rho_m)\rho_\delta & 0 & \rho_m & 0 & (1-\rho_m)\phi_y & (1-\rho_m)\rho_\pi \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & (1-\rho_\delta) & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & -\frac{\omega}{1+\omega\beta} & -\kappa(1+\varphi) & 0 \end{bmatrix} \\
M^2 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\rho_g & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\end{aligned}$$

Transform (1) into:

$$\begin{bmatrix} X_{t+1}^b \\ X_{t+1|t}^f \end{bmatrix} = A^1 \begin{bmatrix} X_t^b \\ X_t^f \end{bmatrix} + A^2 \begin{bmatrix} X_{t|t}^b \\ X_t^f \end{bmatrix} + A^3 u_{t+1} \quad (2)$$

where $A^1 \equiv (M^0)^{-1}M^1$, $A^2 \equiv (M^0)^{-1}M^2$ and $A^3 \equiv (M^0)^{-1}M^3$

Variables that are observable to private agent and relevant for belief updating are summarized

in vector Z_t :

$$Z_t = C \begin{bmatrix} X_t^b \\ X_t^f \end{bmatrix} + v_t \quad (3)$$

or explicitly:

$$\begin{bmatrix} i_t \\ \Delta a_t \\ s_t^p \end{bmatrix} = C \begin{bmatrix} \varepsilon_t^m \\ \hat{g}_t \\ \delta_t \\ \varepsilon_t^\pi \\ \hat{i}_{t-1} \\ \hat{\pi}_{t-1} \\ \hat{y}_t \\ \hat{\pi}_t \end{bmatrix} + \begin{bmatrix} 0 \\ \varepsilon_t^a \\ \varepsilon_t^p \end{bmatrix} \quad (4)$$

where:

$$C = \begin{bmatrix} 1 & (1 - \rho_m)\rho_g & (1 - \rho_m)\rho_\delta & 0 & \rho_m & 0 & (1 - \rho_m)\phi_y & (1 - \rho_m)\rho_\pi \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Take expectation on (2):

$$\begin{bmatrix} X_{t+1}^b \\ X_{t+1}^f \end{bmatrix} = W \begin{bmatrix} X_{t|t}^b \\ X_t^f \end{bmatrix} \quad (5)$$

where $W \equiv A^1 + A^2$. The Schur decomposition of matrix W is HUH^{-1} . Pre-multiply the previous equation by H^{-1} and define $Y_t \equiv H^{-1}X_{t|t}$ we get:

$$Y_{t+1} = UY_t \quad (6)$$

Solving the 2nd block of Y_{t+1} first, where $|\lambda_i| > 1$. Eliminating explosive equilibrium implies:

$$\begin{aligned} Y_{f,t} &= 0 \\ \Rightarrow X_t^f &= GX_{t|t}^b, \end{aligned} \quad (7)$$

where $G \equiv -(H_{22}^{inv})^{-1}H_{21}^{Inv}$. Note that through out the paper the matrix $H \equiv \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} \equiv \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}$, similar notations apply to other matrixes. The 2nd block of (5) together with (7) implies:

$$X_{t+1|t}^b = J^b X_{t|t}^b \quad (8)$$

$$X_{t+1|t}^f = J^f X_{t|t}^b, \quad (9)$$

where $J^b \equiv (W_{11} + W_{12}G)$, $J^f \equiv G(W_{11} + W_{12}G)$.

(2) - (5):

$$\begin{bmatrix} X_{t+1}^b - X_{t+1|t}^b \\ 0 \end{bmatrix} = A^1 \begin{bmatrix} X_t^b - X_{t|t}^b \\ 0 \end{bmatrix} + A^3 u_{t+1}$$

The first block implies that:

$$X_{t+1}^b = H X_t^b + J X_{t|t}^b + A_1^3 u_{t+1} \quad (10)$$

where $J \equiv W_{11} + W_{12}G - A_{11}^1$ and $H \equiv A_{11}$.

Notice that in the belief matrix $X_{t|t}^b$, only two variables are not perfectly observed by agents, those are ε_t^m , and g_t . Denote $x_t \equiv [\varepsilon_t^m, g_t]'$ and $x_{t|t} \equiv [\varepsilon_{t|t}^m, g_{t|t}]'$. The remaining variables are perfectly observable, therefore equal to their counter-parts in matrix X_t^b . Next, we will find law of motion for $x_{t|t}$.

Rewrite (3):

$$\begin{aligned} Z_t &= C_1 X_t^b + C_2 X_{t|t}^f + v_t \\ &= C_1 X_t^b + C_3 X_{t|t}^b + v_t \\ &= C_b X_t^b + M x_{t|t} + v_t, \end{aligned}$$

where $C_3 = C_2 G$, $C_b \equiv C_1 + [0_{3,2} \ C_3(:, 3 : end)]$, $M \equiv C_3(:, 1 : 2)$. The above signal equation can be transform to:

$$Z^x = L x_t + M x_{t|t} + v_t, \quad (11)$$

where $Z^x \equiv Z_t - [0_{3,2} \ C_b(:, 3 : 6)] X_t^b$, $L = C_b(:, 1 : 2)$. Note that:

$$x_t = J^x x_{t-1} + u_t^x, \quad (12)$$

where $J^x = \begin{bmatrix} 0 & 0 \\ 0 & \rho_g \end{bmatrix}$ and $u_t^x = \begin{bmatrix} \varepsilon_t^m \\ \varepsilon_t^g \end{bmatrix}$. Now, results derived in [Svensson and Woodford \(2003\)](#) applies directly.

$$x_{t|t}^b = x_{t|t-1}^b + K(L(x_t - x_{t|t-1}) + v_t) \quad (13)$$

$$K = PL'(LPL' + \Sigma_{vv})^{-1} \quad (14)$$

$$P = J^x[P - PL'(LPL' + \Sigma_{vv})^{-1}LP]J^{x'} + \Sigma_{ux}^2 \quad (15)$$

State Space representation $X_t \equiv \begin{bmatrix} X_t^b \\ x_{t|t} \end{bmatrix}$ and $V_t \equiv \begin{bmatrix} \varepsilon_t^g \\ \varepsilon_t^\delta \\ \varepsilon_t^\pi \\ \varepsilon_t^m \\ \varepsilon_t^a \\ \varepsilon_t^p \end{bmatrix}$. We collect the relevant equations into

one state space representation.

$$X_{t+1} = AX_t + BV_{t+1}, \quad (16)$$

$$X_t^f = FX_t \quad (17)$$

$$X_{t+1|t}^f = F_1X_t \quad (18)$$

with

$$\begin{aligned}
A &= \begin{bmatrix} A_b \\ A_{bb} \end{bmatrix} B = \begin{bmatrix} B_b \\ B_{bb} \end{bmatrix} \\
A_b &= \begin{bmatrix} H & 0_{nb \times 2} \end{bmatrix} + \begin{bmatrix} 0_{nb \times 2} & J(:, 3 : end) & J(:, 1 : 2) \end{bmatrix}, \\
B_b &= A_1^3 L^u, L^u = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \\
A_{bb} &= \begin{bmatrix} 0_{2 \times nb} & J^x - KLJ^x \end{bmatrix} + \begin{bmatrix} KLJ^x & 0_{2 \times 6} \end{bmatrix} \\
B_{bb} &= KLL^x + KL^v, L^v = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, L^x = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
F &= \begin{bmatrix} 0_{nf \times 2} & G(:, 3 : end) & G(:, 1 : 2) \end{bmatrix} \\
F_1 &= \begin{bmatrix} 0_{nf \times 2} & J^f(:, 3 : end) & J^f(:, 1 : 2) \end{bmatrix} \\
F_2 &= \begin{bmatrix} 0_{nf \times 2} & J^{f^2}(:, 3 : end) & J^{f^2}(:, 1 : 2) \end{bmatrix}, J^{f^2} = J^b J^f
\end{aligned}$$

A.1 Term structure

This section derives term structure implied by the model discussed above. The goal is to write down the term structure in terms of our state vector X_t . i_t enters as the fourth element in vector X_{t+1} , therefore:

$$i_t = M_I X_t, \quad (19)$$

where $M_I \equiv A(4, :)$.

The stochastic discount factor, both nominal (M_{t+1}^n) and real (M_{t+1}^r) can be derived from consumer's first order condition:

$$\begin{aligned}
M_{t+1}^n &= \beta \frac{U_{c,t+1} P_t}{U_{c,t} P_{t+1}} \\
M_{t+1}^r &= \beta \frac{U_{c,t+1}}{U_{c,t}}
\end{aligned}$$

Given the utility function specified in this paper, $m_{t+1} \equiv \log M_{t+1}$ is defined as:

$$m_{t+1}^n = \log \beta - y_{t+1} + y_t - \pi_{t+1} + (\rho_\delta - 1)\delta_t - g_t \quad (20)$$

$$m_{t+1}^r = \log \beta - y_{t+1} + y_t + (\rho_\delta - 1)\delta_t - g_t \quad (21)$$

Consider the nominal discount factor, rewrite in terms of state variable:

$$m_{t+1}^n = \log \beta + \begin{bmatrix} -1 & -1 \end{bmatrix} X_{t+1}^f + \begin{bmatrix} 1 & 0 \end{bmatrix} X_t^f + \begin{bmatrix} 0 & -1 & \rho_\delta - 1 & 0_{5 \times 1} \end{bmatrix} X_t$$

Recall that $X_t^f \equiv FX_t$, $X_{t+1}^f = FAX_t + FBV_{t+1}$, plug them in:

$$m_{t+1}^n = \bar{m} + M_n X_t + M_{vn} V_{t+1}, \quad (22)$$

where $\bar{m} \equiv \log \beta$, $M_n \equiv \begin{bmatrix} -1 & -1 \end{bmatrix} FA + \begin{bmatrix} 1 & 0 \end{bmatrix} F + \begin{bmatrix} 0 & -1 & \rho_\delta - 1 & 0_{1 \times 5} \end{bmatrix}$ and $M_{vn} \equiv \begin{bmatrix} -1 & -1 \end{bmatrix} FB$.

Similarly:

$$m_{t+1}^r = \bar{m} + M_r X_t + M_{vr} V_{t+1} \quad (23)$$

where $M_r \equiv \begin{bmatrix} -1 & 0 \end{bmatrix} FA + \begin{bmatrix} 1 & 0 \end{bmatrix} F + \begin{bmatrix} 0 & -1 & \rho_\delta - 1 & 0_{5 \times 1} \end{bmatrix}$ and $M_{vr} \equiv \begin{bmatrix} -1 & 0 \end{bmatrix} FB$.

The no arbitrage condition:

$$P_t^{n+1} = E_t(M_{t+1} P_{t+1}^n) \quad (24)$$

The relation between yield and price of zero coupon bond:

$$i_t^{(n)} = -n^{-1} \log(P_t^n) \quad (25)$$

With (19) (22) (23) (24) (25) we can derive the yield curve as linear function of state variable X_t .

Let's begin with the nominal yield and the real yield curve is derived similarly. Note that $i_t^{(1)} \equiv i_t$.

From (19) (25) and let $n = 1$ we get:

$$\begin{aligned} P_{t+1}^1 &= \exp(-i_{t+1}) \\ &= \exp(-M_I X_{t+1}) \\ &= \exp(-M_I A X_t - M_I B V_{t+1}) \end{aligned} \quad (26)$$

Plug (23) and (26) into (25):

$$\begin{aligned}
P_t^2 &= v_{h,t}(M_{t+1}P_{t+1}^1) \\
&= v_{h,t}[\exp(\bar{m} + M_n X_t + M_{vn} U_{t+1} - M_I A X_t - M_I B V_{t+1})] \\
&= v_{h,t}[\exp(\bar{m} + (M_n - M_I A) X_t + (M_{vn} - M_I B) V_{t+1})]
\end{aligned}$$

Under the assumption that shocks are normally distributed, the previous equation follows a log-normal distribution¹⁸. Therefore:

$$P_t^2 = \exp(\bar{m} + (M_n - M_I A)DX_t + 0.5Var_2), \quad (27)$$

we have used the fact that $X_t|t = DX_t$ with $D \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$. Var_2 depends on

parameters and are remained to be determined. By (25)

$$\begin{aligned}
i_t^{(2)} &= -0.5 \log(P_t^2) \\
&= -0.5\bar{m} - 0.25Var_2 - 0.5(M_n - M_I A)DX_t
\end{aligned} \quad (28)$$

We have derived the expression for yield of a zero-coupon bond with maturity of 2. We will show in general for $n \geq 2$:

$$\log P_t^n = A_n + B_n X_t \quad (29)$$

For $n = 1$, $\log P_t^1 = -M_I X_t$. And we have shown for $n = 2$:

$$\begin{aligned}
A_2 &= \bar{m} + 0.5Var_2 \\
B_2 &= (M_n - M_I A)D
\end{aligned}$$

¹⁸If x follows a log-normal distribution, $E(e^x) = e^{E(x) + 0.5\sigma_x^2}$

In order to find the general rule, let's take one more step and set $n = 3$:

$$P_t^3 = v_{h,t}(M_{t+1}P_{t+1}^2)$$

(29) implies that $P_{t+1}^2 = \exp(A_2 + B_2AX_t + B_2BV_{t+1})$. Therefore:

$$\begin{aligned} P_t^3 &= v_{h,t}[\exp(\bar{m} + M_nX_t + M_{vn}V_{t+1} + A_2 + B_2AX_t + B_2BV_{t+1})] \\ &= v_{h,t}[\exp(\bar{m} + A_2 + (M_n + B_2A)X_t + (M_{vn} + B_2B)V_{t+1})] \end{aligned}$$

Hence:

$$\log P_t^3 = A_3 + B_3X_t,$$

with

$$\begin{aligned} A_3 &= \bar{m} + A_2 + 0.5Var_3 \\ B_3 &= (M_n + B_2A)D \end{aligned}$$

The recursive relations for A_n and B_n are:

$$A_n = A_{n-1} + \bar{m} + 0.5Var_n \quad (30)$$

$$B_n = (M_n + B_{n-1}A)D \quad (31)$$

The expressions for $Vars$ are:

$$Var_2 = [B_2D^{-1}]^b P [B_2D^{-1}]^{b'} + (M_{vn} + M_I B) \Sigma_{VV} (M_{vn} + M_I B)' + \sigma^2 \Psi^2 (\sigma_g^2 + \sigma_a^2 + \sigma_m^2)$$

$$Var_3 = [B_3D^{-1}]^b P [B_3D^{-1}]^{b'} + (M_{vn} + B_2B) \Sigma_{VV} (M_{vn} + B_2B)' + \sigma^2 \Psi^2 (\sigma_g^2 + \sigma_a^2 + \sigma_m^2)$$

$$Var_n = [B_nD^{-1}]^b P [B_nD^{-1}]^{b'} + (M_{vn} + B_{n-1}B) \Sigma_{VV} (M_{vn} + B_{n-1}B)' + \sigma^2 \Psi^2 (\sigma_g^2 + \sigma_a^2 + \sigma_m^2)$$

Thus the yield curve with different maturities can be collected in:

$$\begin{bmatrix} i_t \\ i_t^2 \\ \cdot \\ \cdot \\ i_t^n \end{bmatrix} = \begin{bmatrix} -A_1 \\ -\frac{1}{2}A_2 \\ \cdot \\ \cdot \\ -\frac{1}{n}A_n \end{bmatrix} + \begin{bmatrix} -B_1 \\ -\frac{1}{2}B_2 \\ \cdot \\ \cdot \\ -\frac{1}{n}B_n \end{bmatrix} X_t + e_t^i \quad (32)$$

B Proof of Proposition 1

Proof: At time t a positive monetary shock leads to a positive monetary surprise, thus the agent updates $g_{t|t}$ upwards as little as possible using $\sigma_{m,t} = \bar{\sigma}_m$. Note that $E_t(i_{t+1})$ is positively related to $g_{t|t}$ with a coefficient θ_g .

At time $t + 1$, the misinterpretation committed at time t leads to a negative monetary surprise $-\theta_g g_{t|t}$, thus the agent updates $g_{t+1|t+1}$ downwards. The exact amount of forecast revision depends on her choice of $\sigma_{m,t}$ and $\sigma_{m,t+1}$. It is apparent that $\sigma_{m,t+1} = \underline{\sigma}_m$ leading to a maximum downward revision of $g_{t+1|t+1}$ (worse case) independently on the choice of $\sigma_{m,t}$.

The $\sigma_{m,t}$ chosen at $t + 1$ remains the same as before, i.e. $\bar{\sigma}_m$. To see this, by contradiction assume that the agent revises her choice of $\sigma_{m,t}$ downward such that $g_{t|t+1} - g_{t|t} = \Delta$. As a result, $g_{t+1|t+1}$ would be $(1 - (k_{t,t} - k_{t,t+1})\theta_g)\Delta$ higher than the one associated with $\sigma_{m,t} = \bar{\sigma}_m$, thus this is not the worse case – contradicts the agent is a max-minimizer. Note that I have used the fact that under the worse case choice of $\sigma_{m,t}$ the belief updating equation is $g_{t+1|t+1}^* = g_{t|t} - k_{t,t}\theta_g g_{t|t}$ and under the alternative case $g_{t+1|t+1} = g_{t|t+1} - k_{t,t+1}\theta_g g_{t|t+1}$, with $(k_{t,t} - k_{t,t+1}) < 0$.

Similarly, at time $t + j$ for $j > 1$, the agent keeps being shocked by negative monetary surprise until she fully learned the truth, i.e there was merely a pure monetary shock at t . And the worse case belief is associated with $\underbrace{\{\bar{\sigma}_m, \dots, \underline{\sigma}_m\}}_{j \text{ times}}$.

The same arguments hold for a negative monetary shock Proposition 2. The choice of $\sigma_{m,t}$ made at time t is not revised at future periods. ■

C Data

C.1 Construction of factors

The goal is to construct factors (principle components) that represent state of the economy but are orthogonal to the monetary instrument. Many DSGE model, such as the one presented in this paper, has the solution that takes the following form:

$$Y_t = FX_t + e_t,$$

where X_t is a N by T vector that includes all state variables that evolve exogenously and Y_t is a M by T vector that contains observable variables. In practice M is large and in theory N is small. Using the realtime 110 monthly macroeconomics time series from FRED-MD, I construct the first

N principle components denoted as $F_{all,t}$ with N equal to five for the baseline regressions and N equal to eight for the robustness check represented in the paper. I have used realtime data, i.e $F_{all,t}$ are constructed using data up to time t, in order to excluding future information. Those principle components are good representation of the state of the economy (X_t). However since those will be used to "clean" (as controls) the HFI monetary surprises, it is important to remove the state variable related to monetary shock from $F_{all,t}$. To this end, I follow [Bernanke, Boivin and Eliasch \(2004\)](#)'s approach. I construct the first N principle components from slow-moving variables (those do not response to monetary shock on impact) denoted as $F_{slow,t}$, then estimate regression of the following form:

$$F_{all,t} = c + \beta_1 F_{slow,t} + \beta_1 FFR_t + \beta_2 Y2Y_t + u_t \quad (33)$$

where FFR_t and $Y2Y_t$ denote the fed funds rate and two year nominal yields: those are the proxies for monetary instruments. This regression aims to determine the part in $F_{all,t}$ that originate from monetary shocks. $F_{slow,t}$ is included to ensure unbiased estimates for β_1 and β_2 . The desired factors F_t are thus constructed as the difference between the $F_{all,t}$ and those originate from monetary shocks:

$$F_t = F_{all,t} - \hat{\beta}_1 FFR_t - \hat{\beta}_2 Y2Y_t. \quad (34)$$

In [Loria et al. \(2017\)](#), we show that the constructed F_t have predictive power on the HFI monetary surprises suggesting the information asymmetry between the central bank and private agent. And it is thus important to control for F_t in order to identify the pure monetary shocks from the HFI monetary surprises.

D Bayesian Estimation

Measurement equations The measurement equations are:

$$\log(RGDP_t/POP_t) - \log(RGDP_{t-1}/POP_{t-1}) = g^* + \hat{y}_t - \hat{y}_{t-1} + \hat{g}_t + \varepsilon_t^a$$

$$\text{Inflation}_t = \hat{\pi}_t + \pi^*$$

$$Y2Y_t/4 = \hat{i}_t + e_t^{y2}$$

$$Y5Y_t/4 = -\frac{1}{20}B_{20}X_t + e_t^{y5}$$

$$Y10Y_t/4 = -\frac{1}{40}B_{40}X_t + e_t^{y10},$$

where $RGDP_t/POP_t$ denotes the per capita real GDP, Inflation is the GDP price deflator, $Y2Y_t$, $Y5Y_t$ and $Y10Y_t$ are the demeaned nominal yields with maturity of two, five and ten years respectively. By demeaning the yield I forgo testing the model's ability to match the average slope of yield curve. Matching the empirical slope of the yield curve would require additional features such as liquidity premium, which is beyond the scope of this paper. [Nimark \(2008\)](#) take the same data transformation. Similar to the empirical exercises conducted above, I have used the quarterly return of the two years nominal yield as policy instrument. This is a short-cut to allow me to interpret the monetary shock in the model as a mix of conventional monetary shock (shock to the fed funds target) and the forward guidance shock (shock to interest rate at longer horizon). Moreover, using two years yields as policy instrument makes estimation using data beyond 2008 (zero lower bound) possible¹⁹. Note that I have add measurement errors to the yields to allow for potential time varying risk premium that are not captured in the model.

Posterior Convergence Check

¹⁹However, results are robust if the two years nominal yields enter in the measurement equation as yield with maturity of eight periods.

Figure 28: Posterior Distributions

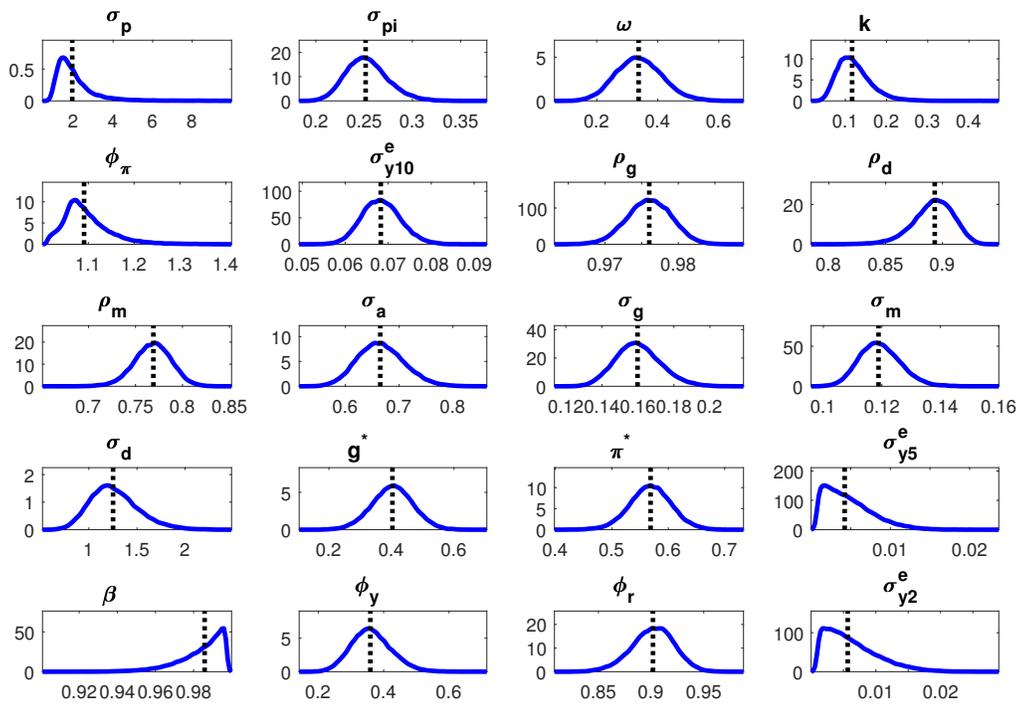
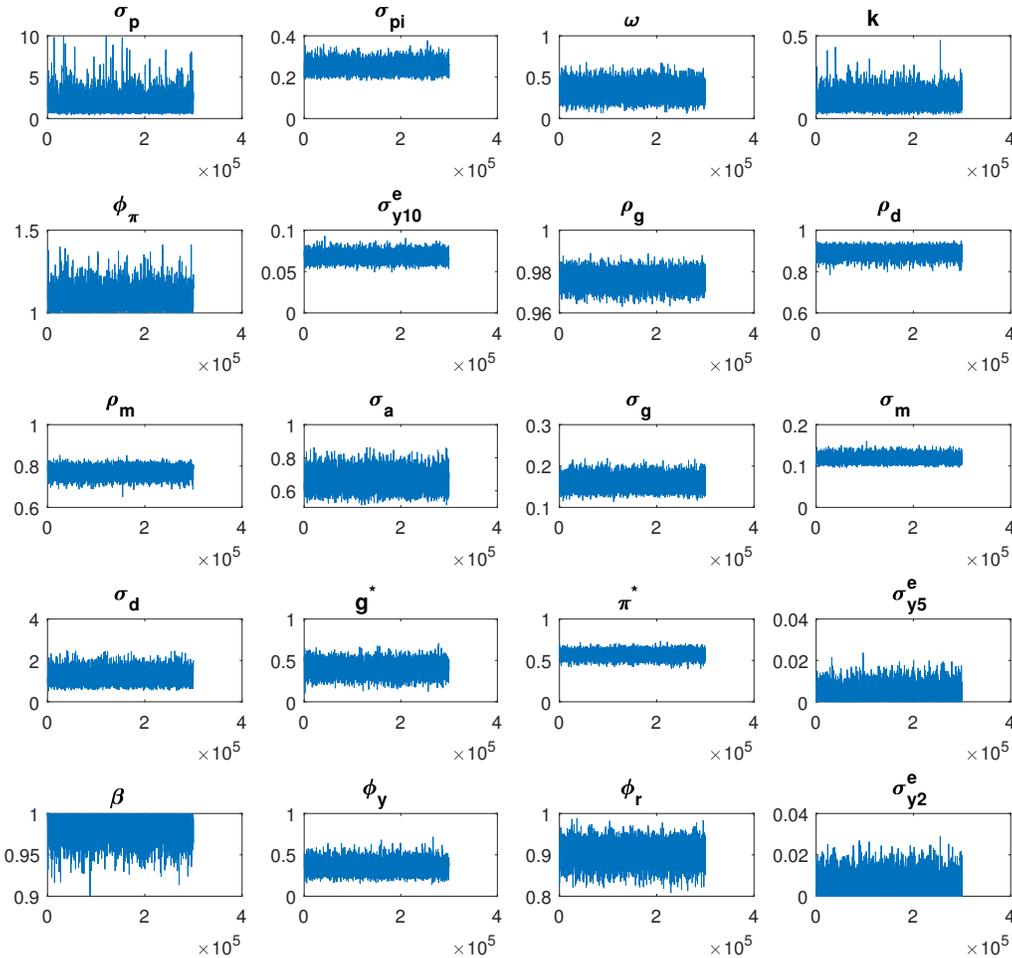


Figure 29: Convergence Check I: Traceplots



Note: A traceplot is a plot of the value of the draw of the parameter at each iteration against the iteration number

Figure 30: Convergence Check II: Recursive Average

